

COPULA BASED PREDICTIONS IN SMALL AREA ESTIMATION FOR BETA DISTRIBUTED RESPONSE VARIABLES

DIN NURIKA AGUSTINA



STATISTICS AND DATA SCIENCE
SCHOOL OF DATA SCIENCE, MATHEMATICS, AND INFORMATICS
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RINGKASAN

DIN NURIKA AGUSTINA. Pendekatan Copula dalam Pendugaan Area Kecil untuk Peubah Respon Bersebaran Beta. Dibimbing oleh Prof. Dr. Ir. Khairil Anwar Notodiputro, MS, Dr. Ir. Indahwati, M.Si, dan Dr. Utami Dyah Syafitri, S.Si. M.Si.

Metode pendugaan berbasis model, yaitu pendugaan area kecil (*small area estimation*), merupakan metode alternatif untuk mengatasi permasalahan keterbatasan jumlah contoh atau contoh berukuran kecil ketika metode pendugaan langsung digunakan dalam suatu survei. Ketidacukupan jumlah contoh menyebabkan hasil pendugaan langsung memiliki presisi rendah, sehingga tidak memenuhi standar untuk statistik yang akurat. Metode pendugaan area kecil hadir untuk menjawab tantangan ini, dengan menghasilkan dugaan parameter yang andal, meskipun terdapat kendala dalam kecukupan ukuran contoh.

Penelitian terkait pendugaan area kecil mengalami perkembangan signifikan, ditandai dengan munculnya berbagai inovasi baik di Indonesia maupun secara global. Berbagai inovasi tersebut mencakup pengembangan model pendugaan area kecil yang mengakomodasi beragam sebaran data, pendekatan spasial, penggunaan pembelajaran mesin, dan pengembangan model dengan peubah penyerta yang mengakomodir kesalahan pengukuran, serta pendekatan fungsi kumulatif bersama atau copula dalam pemodelan. Semua inovasi ini dirancang untuk meningkatkan akurasi dan presisi dugaan dari pendugaan area kecil.

Keunggulan pendekatan copula dalam pendugaan area kecil telah ditunjukkan dalam beberapa penelitian sebelumnya, khususnya melalui pengembangan model multivariat copula yang dapat ditukar (*multivariate exchangeable copula*). Model multivariat copula yang dapat ditukar merupakan salah satu inovasi terkini dalam pendugaan area kecil yang berhasil meningkatkan akurasi dugaan, terutama untuk data yang mengikuti sebaran normal dengan kondisi bentuk sebaran yang menjulur (*generalized normal*). Meskipun demikian, pengembangan lebih lanjut terkait peningkatan akurasi dan presisi melalui pendekatan copula untuk pendugaan sisaan model masih relatif terbatas dalam literatur, terutama dalam konteks model pendugaan area kecil.

Data bersebaran beta pada praktiknya merupakan sebaran kontinu dengan nilai berada pada selang terbuka di antara 0 (nol) sampai 1 (satu). Peubah bersebaran beta bisa berupa proporsi yang berasal dari data diskrit maupun kontinu, persentase, rasio, maupun indeks. Berbagai contoh indikator bersebaran beta antara lain persentase perkawinan anak, proporsi pengeluaran biaya kesehatan terhadap pengeluaran total (*out of pocket health expenditure*) oleh rumah tangga, persentase kemiskinan, persentase putus sekolah, persentase pengangguran, rasio Gini, dan sejenisnya.

Ide kebaruan disertasi ini adalah mengembangkan model pendugaan area kecil bagi peubah respon bersebaran beta dengan pendekatan copula untuk pendugaan sisaan model yang selanjutnya disebut sebagai model pendugaan area kecil beta copula (*small area estimation beta copula*/SAE BC). Model SAE BC merupakan inovasi baru dalam pendugaan area kecil level unit, untuk peubah respon bersebaran beta dengan beragam bentuk sebaran data, terutama bentuk sebaran data dengan karakteristik menjulur disertai adanya autokorelasi. Model



SAE BC dengan pendekatan copula dalam pendugaan sisaan model merupakan sudut pandang baru dalam meningkatkan akurasi pendugaan area kecil, berbeda dari metode konvensional yaitu penambahan efek acak dalam model. Evaluasi model SAE BC yang disajikan dalam disertasi ini juga memperkenalkan aspek kebaruan lokalitas menggunakan data empiris untuk menduga persentase perkawinan anak di setiap kecamatan di Provinsi Jawa Barat, Indonesia, 2021.

Kajian pertama disertasi merupakan evaluasi empiris terhadap model multivariat copula yang dapat ditukar sebagai model rujukan disertasi. Aspek kebaruan lokalitas kajian terletak pada penggunaan data empiris untuk pendugaan rata-rata lama sekolah pemuda di tingkat kecamatan di Provinsi Jawa Barat, Indonesia, 2021. Evaluasi kinerja model multivariat copula yang dapat ditukar dibandingkan dengan model dasar pendugaan area kecil yaitu model regresi galat tersarang (*nested error regression*). Hasil evaluasi menunjukkan bahwa bentuk sebaran peubah memiliki pengaruh dalam pemilihan pemilihan model. Penelitian-penelitian pada kondisi sebaran respon normal dengan bentuk sebaran menjulur (*generalized normal*) menunjukkan model multivariat copula yang dapat ditukar lebih unggul. Selanjutnya, temuan kajian ini menunjukkan regresi galat tersarang memiliki performa lebih baik dibanding model multivariat copula yang dapat ditukar pada kondisi sebaran peubah respon normal simetris.

Hasil kajian pertama mengenai pengaruh bentuk sebaran terhadap kinerja model menjadi dasar perancangan simulasi pada kajian kedua untuk mengevaluasi model SAE BC. Simulasi dirancang menghasilkan peubah respon berdistribusi beta dengan variasi bentuk sebaran (menjulur dan simetris) serta tingkat autokorelasi (rendah dan tinggi). Model SAE BC dibangun dalam kerangka regresi linier dengan transformasi logit, di mana estimasi parameter dilakukan menggunakan optimisasi nonlinier berbasis distribusi beta (SAE BC Beta) dan metode kuadrat terkecil (SAE BC OLS). Dugaan sisaan kemudian diintegrasikan ke dalam model menggunakan pendekatan copula. Evaluasi dilakukan dengan perbandingan terhadap model SAE campuran logistik yang memiliki karakteristik frekuentif serupa. Hasil menunjukkan bahwa model-model SAE BC secara konsisten memberikan kinerja lebih baik dalam hal akurasi dan presisi, untuk berbagai bentuk distribusi beta dan tingkat autokorelasi.

Kajian ketiga berfokus pada evaluasi kinerja model SAE BC dibandingkan dengan model SAE campuran logistik, dengan memperhatikan aspek kebaruan lokalitas. Aspek kebaruan lokalitas adalah menggunakan data empiris untuk pendugaan persentase perkawinan anak di tingkat kecamatan di Provinsi Jawa Barat pada tahun 2021. Hasil kajian ketiga adalah persentase perkawinan anak menurut kecamatan di Provinsi Jawa Barat dengan model SAE BC Beta sebagai model yang memiliki kinerja lebih baik untuk data perkawinan anak dengan karakteristik bersebaran beta dan menjulur.

Kata kunci: copula, kesetaraan gender, level unit, pendidikan berkualitas, pendugaan area kecil, sebaran beta.

SUMMARY

DIN NURIKA AGUSTINA. Copula Based Predictions in Small Area Estimation for Beta Distributed Response Variables. Supervised by Prof. Dr. Ir. Khairil Anwar Notodiputro, MS, Dr. Ir. Indahwati, M.Si, and Dr. Utami Dyah Syafitri, S.Si, M.Si.

The model-based estimation approach, specifically small area estimation (SAE), serves as an alternative method to address the challenges posed by limited sample sizes or small sample when direct estimation techniques are employed in surveys. Insufficient sample size can result in low precision in direct estimation outcomes, failing to meet the required standards for producing accurate statistics. SAE was introduced to overcome this issue by providing reliable parameter estimates, even when sample size limitations exist.

Research on SAE has significantly advanced, with numerous innovations emerging both in Indonesia and globally. These innovations include the development of SAE models that account for various data distributions, spatial approaches, machine learning techniques, and models that incorporate auxiliary variables to handle measurement errors, as well as the application of cumulative distribution functions or copulas in SAE modeling. All of these innovations aim to improve the precision and accuracy of SAE estimates.

The benefits of using copula approaches in SAE have been demonstrated in several prior studies, particularly through the development of the multivariate exchangeable copula model. The multivariate exchangeable copula model represents a recent innovation in small area estimation that has notably enhanced the accuracy of estimates, especially for data following a normal distributed response variable with a skewed distributional shape (generalized normal). However, further development regarding accuracy improvements through copula approaches for model residual estimation remains relatively limited in the literature, especially within the context of small area estimation modeling.

The beta-distributed data, in practice, represent a continuous distribution with values lying within the open interval between 0 (zero) and 1 (one). Variables following a beta distribution may include proportions derived from either discrete or continuous data, as well as percentages, ratios, or indices. Common examples of beta-distributed indicators include the percentage of child marriages, the proportion of out-of-pocket health expenditure relative to total household expenditure, the poverty rate, school dropout rate, unemployment rate, the Gini ratio, and similar indicators.

The innovative contribution of this dissertation lies in developing a small area estimation model for beta-distributed response variables using a copula approach for residual estimation, referred to as the small area estimation beta copula (SAE BC) models. The SAE BC models represents a novel contribution to small area estimation methods at the unit level, particularly for beta-distributed response variables characterized by skewness and autocorrelation. The SAE BC models, incorporating a copula approach for residual estimation, provides a fresh perspective on enhancing the accuracy of small area estimates, distinct from conventional methods, such as the inclusion of random effects in the model. The evaluation of the SAE BC models presented in this dissertation also introduces a



novel locality aspect by using empirical data to estimate child marriage percentages in each sub-district in West Java Province, Indonesia, in 2021.

The first part of this dissertation presents an empirical evaluation of the exchangeable multivariate copula model as the reference model. The local novelty of this study lies in the use of empirical data to estimate the youth mean years of schooling at the sub-district level in West Java Province, Indonesia, in 2021. The performance of the exchangeable multivariate copula model is compared with that of a baseline small area estimation model, specifically the nested error regression model. The evaluation results indicate that the distributional characteristics of the response variable play a critical role in model selection. When the response variable follows a generalized normal distribution exhibiting skewness, the exchangeable multivariate copula model outperforms the nested error regression model. Conversely, under a symmetric normal distribution, the nested error regression model demonstrates superior performance compared to the copula-based model. These findings highlight the importance of considering the distributional form of the response variable when selecting the most appropriate small area estimation method.

The findings from the first study regarding the impact of distributional shapes on model performance served as the basis for the simulation design in the second study, which aimed to evaluate the SAE BC model. The simulations were designed to generate beta-distributed response variables with varying distributional shapes (skewed and symmetric) and levels of autocorrelation (low and high). The SAE BC model was developed within a linear regression framework with a logit transformation. Parameter estimation was carried out using nonlinear optimization based on the beta distribution for the SAE BC Beta model, and the ordinary least squares (OLS) method for the SAE BC OLS model. The residuals estimation were then integrated into the modeling framework using a copula-based approach. Model performance was evaluated by comparing the SAE BC models with the logistic mixed SAE model, which shares similar frequentist characteristics. The results demonstrated that the SAE BC models consistently outperformed the logistic mixed model in terms of both accuracy and precision across various beta distribution shapes and autocorrelation levels.

The third study focuses on the performance evaluation of the SAE BC models in comparison to the mixed logistic SAE model, with an emphasis on the locality novelty. The locality novelty in this study is the use of empirical data to estimate child marriage percentages at the sub-district level in West Java Province in 2021. The findings of the third study reveal that the SAE BC Beta model outperforms other models for estimating child marriage percentages at the sub-district level in West Java, particularly for data with beta distribution characteristics, skewness, and autocorrelation.

Keywords: beta distribution, copula, gender equality, quality education, small area estimation, unit level.



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on Statistics and Data Science

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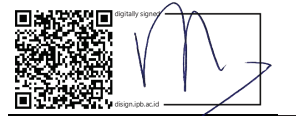
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