Developing Procedures for Optimization of Tank Model's Parameters

Budi I. Setiawan[1], T. Fukuda[2] and Y. Nakano[2]

[1]Professor. Center for Research on Engineering Application in Tropical Agriculture (CREATA), Bogor Agricultural University, PO BOX 220, Bogor 16002, Indonesia, E-mail: budindra@ipb.ac.id; and budindra@telkom.net.

[2]Associate Professor and Professor. Department of Bio-production and Environmental Sciences, Kyushu University, 6-10-1 Hakozaki, Higashi-ku, Fukuoka-shi, 812-8581, Japan

Abstract. Tank Model is one of the hydrological models for analyzing the characteristics of river flow. The model can give information of water availability and be used to predict flood occurrences. As it is *commonplace*, this model needs calibration, and it is usually done by setting the embodied parameters. In the form of Standard Tank Model, the number of parameters accounts to 12. Many optimization methods have been recognized but so far there is no single method available for general application. This paper introduces an optimization technique to determine the parameters with taking into account the conformity to water balance in addition to the best fitting and some error indicators. Here, two data from Terauchi Watershed in Japan and Ciriung Watershed in Indonesia were used for clarification, which show that this optimization technique gained fast and accurate results. This technique has been made available to use in the form of an application software and openly possible to accommodate the other forms of Tank Model.

Keywords. Hydrological model, tank model, parameters, optimization, and software.

Background

Various attempts to study dynamic water balance in a watershed have widely been conducted and some has resulted in good hydrological models. Among others, there is a tank model that divides the watershed into several reservoirs and waterflow regimes (Sugawara *et.al*., 1986). Originally, a Standard Tank Model consists of 4 (four) reservoirs and 5 (five) ouflows (Fig. 1). In further developments, we can find a set of tank models constructed in many different ways trying to approach the field conditions (Elhassan *et.al.,* 2001).

Currently, the challenges lay on how to determine the embodied parameters of Tank Model. The commonly used optimization technique is hard to be applied since mathematical formulations of Tank Model involve many discrete functions. There are various optimization techniques have been developed specifically (Kadoya and Tanakamaru 1981; Fujihara *et.al.*, 2001). In most cases, a designer of Tank Model has to develop his own optimization technique, in which some of them even still use eye-fitting. In any cases, they might raise problems over the acceptance of resulted parameters.

Commonly, the degree of satisfaction determined by the coefficient of determination and some error indicators which compare discharge data and calculation results of Tank Model. We notice, however, attention is less given to observe the conformity of calculated water balance and physical aspects of the fields under consideration. With solely looking after the best-fitted parameters, It is not uncommon to see the resulted parameters then causing water level in a specific reservoir beyond its capacity to store the water. Some even came to a conclusion that the designed Tank Model had failed after learning that the sought parameters could give preferable satisfactions, and then insisted to reconstruct another structure of the model.

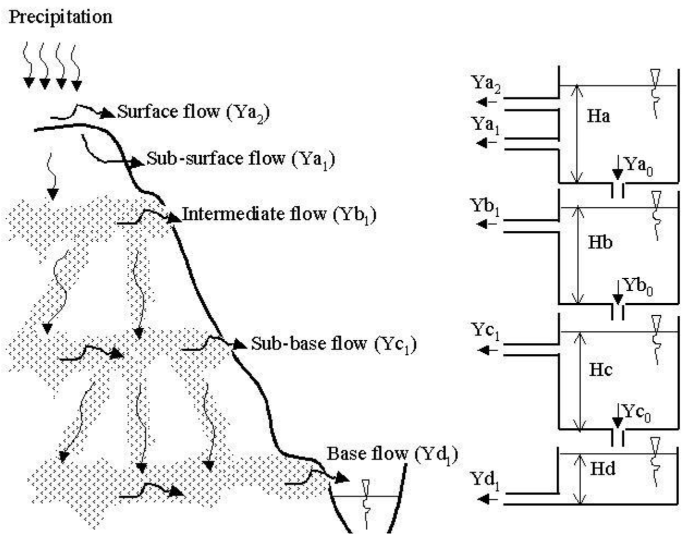

Figure 1. Schematic Standard Tank Model

So, it is of interest to develop a standard technique of optimization with not only looking for best conformity but also more or less capable to approach the physical realities and/or boundaries of the field under study. The results of the technique might give to a justification on whether the model has succeeded or should be reconstructed or modified furthermore.

The objectives of this work are 1) to develop procedures of optimization technique of Tank Model's parameters, and 2) to construct an application program that hopefully can be referred by users of Tank Models as a standard optimization technique.

Brief Description of Tank Model

Figure 1 shows the Standard Tank Model and hypothetical water regimes in a watershed. This model is constructed of four vertical reservoirs, of which from top to bottom parts represents the Surface Reservoir (A), Intermediate Reservoir (B), Sub-base Reservoir (C), and Base Reservoir (D). In this concept, water can fill the underneath reservoir, and can go reversibly if evapotranspiration is so predominant. The horizontal outlet reflects the outflow, consisting of Surface Flow (Ya_2), Subsurface Flow (Ya_1), Intermediate Flow (Yb_1), Sub-base Flow (Yc_1), and Base Flow (Yd_1). Each outflow only occurs when the water level at each reservoir (Ha, Hb, Hc and Hd) is higher than its outlet (Ha_1, Ha_2, Hb_1 and Hc_1). The outflow at each outlet is also influenced by the characteristics of the outlet, i.e., A_0, A_1, A_2, B_0, B_1, C_0, C_1, and D_1, which further are called as the parameters of the Tank Model to be determined. All in all there are 12 (twelve) parameters.

Globally, the water balance equation can be written as follows:

$$\frac{dH}{dt} = P(t) - ET(t) - Y(t) \quad (1)$$

Where, H is water level (mm), P is precipitation (mm/day), ET is evapotranspiration (mm/day), Y is total flow (mm/day), and t *is* time (day).

The total flow is the summation of the flow components that can be written as follows:

$$Y(t) = Ya(t) + Yb(t) + Yc(t) + Yd(t) \quad (2)$$

The water balance in each reservoir in more detail can be written as:

$$\frac{dHa}{dt} = P(t) - ET(t) - Ya(t) \quad (3)$$

$$\frac{dHb}{dt} = Ya_0(t) - Yb(t) \quad (4)$$

$$\frac{dHc}{dt} = Yb_0(t) - Yc(t) \quad (5)$$

$$\frac{dHd}{dt} = Yc_0(t) - Yd(t) \quad (6)$$

Where, Ya, Yb, Yc and Yd are the horizontal flow components from each reservoir, and Ya_0, Yb_0 and Yc_0 are the vertical components.

Practically, the total outflow (y) is frequently considered as the accumulation of the outflows from a system of water region. In a watershed, the total outflow can represent a river discharge, and in a paddy field can be considered as drainage flow. In reality, it is very difficult or may be impossible to find out or distinguish the other outflows. It is clear that Tank Model only represent the study area globally. Commonly, the total outflow is the target to check out the workability of a Tank Model. Observation focused on the maximum values of the total outflow (Y) is needed in the study of flood. Meanwhile, information about minimum values or Base Flow (Yd_1) will be useful for the water utilization planning, such as irrigation, fishery and so on, particularly during the dry periods. Thus, focus of observation might also be important in the determination of optimized parameters.

Optimization Technique

It is of interest to design an optimization technique capable of facilitating each structure of the Tank Model. Such optimization technique must not require detail information in every Tank Model. Herewith, the Tank Model is assumed as one Black Box and its behavior is observable when receiving updated parameters. We applied Marquardt algorithm, which in a simple case it is very quick and effective in finding the optimum parameter even for extremely non-linear equations (Marquardt, 1963; Setiawan and Shiozawa, 1992). Aside from that, it also has been equipped with maximum and minimum values for each parameter. Considering the basic structure of a Tank Model, the algorithm is reconstructed so it can receive the data of total outflow and calculation results from the model. With the Tank Model having 4 (four) reservoirs, an algorithm in the form of **procedure** is constructed the following considerations:

1) It has an input argument for receiving all parameters (**B**);

2) It has an input argument for receiving net of rainfall data minus evapotranspiration (**X**); and;
3) It has an output argument for transferring the calculation outflow of the Tank Model (**Yc**).

Herewith, in Pascal language, the **procedure** can be written as follows:

```
procedure TankModel(B:ArrayM; X:single; var Yc:single);
begin
…
end;
```

The complete writing of the program can be seen in Appendix 1. Inside this **procedure**, embodied (4) four **procedure**, each is used for calculating outflow from every reservoir. And, in the **main program,** the water level at each reservoir is calculated.

The Marquardt[1] algorithm is constructed with the following considerations:

1) It has an input argument for receiving Minimum Value (**Bmin**) and Maximum Value (**Bmax**) parameters;
2) It has an input argument for receiving net of rainfall and evapotranspiration (**X**);
3) It has an input argument for receiving debit data (**Yd**); and
4) It has an input/output argument for receiving and transferring parameter (**B**).

Herewith, the designed **procedure Marquardt** is written as:

```
procedure Marquardt (Bmin,Bmax:ArrayM; X,Yd:ArrayN; var B:ArrayM);
begin {Main of Marquardt}
…
end;{End of Marquardt}
```

The **procedure Marquardt** consists of **procedure derivative** with the function to carry out first derivation of the Tank Model numerically, **procedure least square** for minimizing error, and **procedure gauss** for the calculation of the renewed parameters.

By giving initial approximation of the parameters, iteration process continued until the sum of absolute changes of updated parameters reached a tolerable value, good conformity and less discrepancy of water balance. The last updated parameters then are considered as the final solutions.

Materials and Method

[1] Due to the limited space of pages, procedure Marquardt cannot be presented here, but can be directly accessed from the first author for free of charge.

Table 1. Initial Parameters

Parameter	Initial	Min	Max
A_0	0.2	0	1
A_1	0.1	0	1
A_2	0.1	0	1
B_0	0.06	0	1
B_1	0.03	0	1
C_0	0.012	0	1
C_1	0.006	0	1
D_1	0.001	0	1
Ha_1	15	5	15
Ha_2	25	25	60
Hb_1	15	0	30
Hc_1	15	0	60

Here, the Optimization Technique was tested to determine the parameters of the Standard Tank Model in 2 (two) watersheds, i.e., Terauchi Watershed (5055 ha) in Fukuoka, Japan, and Ciriung Watershed (120 ha) in Banten Province, Indonesia. From the Terauchi Watershed, daily data of rainfall, evapotranspiration, and water discharge were well recorded for 10 (ten) years, from 1986 through 1995 (Fukuda and Nakano, 2001). And, daily data from Ciriung Watershed were collected in 2002/2003 (Suprayogi*,* 2003).

To start the optimization process, we used initial values of parameters introduced by Sugawara *et.al*. (1986), such as seen in Table 1. The table also shows the suggested Minimum Value and Maximum Value of each parameter. It was also necessary to give initial values of water level (*Ha, Hb, Hc* dan *Hd*) in each reservoir. Actually, the values were somewhat difficult to determine and even impossible to obtain from the field. Therefore, initial approximation was then done before the iteration process by giving a value of $Hd=Yd/D_1$, i.e., when the *Base Flow* was at the lowest while other flow components were assumed zero (*i.e.,* during the dry season). Subsequently, calculation was done to produce new estimations for all water level values (*Ha, Hb, Hc* dan *Hd*) by checking out the water balance in the first year. If the water balance was good enough (minimum discrepancy), the water level value obtained was used as the initial condition to start the optimization process.

Updating all parameter values for each iteration was done was terminated after the absolute total of the changes in all parameters was less than the given tolerance value, i.e., 0.00001. The successfulness of the optimization technique was shown using coefficient of correlation (*R)* and 7 (seven) error indicators, i.e., 1) *Root Mean Square Error* (*RMSE*), 2) *Mean Absolute Error* (*MAE*), 3) *Logarithmic RMSE* (*LOG*), 4) Standard χ, 5) Squared Standard χ^2; 6) Relative Error (*RE*), and 7) Squared Relative Error (*RR*), which are written as the following equations (Fujihara, et.al., 2001):

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Qc_i - Qo_i)^2} \quad \text{(7)}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|Qc_i - Qo_i\right| \quad \text{(8)}$$

$$LOG = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\log Qc_i - \log Qo_i)^2} \quad \text{(9)}$$

$$\chi = \frac{1}{N}\sum_{i=1}^{N}\frac{\left|Qc_i - Qo_i\right|}{\sqrt{Qo_i}} \quad \text{(10)}$$

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{(Qc_i - Qo_i)^2}{Qo_i} \quad \text{(11)}$$

$$RE = \frac{1}{N}\sum_{i=1}^{N}\frac{\left|Qc_i - Qo_i\right|}{Qo_i} \qquad (12)$$

$$RR = \frac{1}{N}\sum_{i=1}^{N}\frac{\left(Qc_i - Qo_i\right)^2}{Qo_i^{\,2}} \qquad (13)$$

Results and Discussion

Table 2 shows the optimum parameter and error indicator after going through optimization process for both watersheds, i.e., Terauchi and Ciriung. At both watersheds, the Standard Tank Model resulted in very good water balance where the occurred discrepancy approaching zero. As such, the coefficient of correlation (***R***) was over 0.9, whilst the other error indicator was almost less than one except for ***RMSE*** which was over 3 and 1, repectively.

Hydrographic for Terauchi River in 1996 can be seen in Figure 2. The Standard Tank Model seems successful in calculating low discharges but less successful in approaching the maximum descharges. Similar phenomenon was also shown in the hydrographic for the previous years up to 1995. Here, ***RMSE*** is always higher to others. Even as seen in Figure 3, the ***RMSE*** in 1991 reaches 8. Nevertheless, almost all errors in that year were actually enlarging as compared to those in the previous and following years.

Figure 4 shows the hydrographic for Ciriung River. Different from Terauchi River, Ciriung River in the early year shows somewhat high discharges. But in the middle of the year starting from the end of June up to the early December of 2002 (dry season), it shows a decreasing discharges. Here, the Standard Tank Model seems to be successful in approaching the peak discharges as compared to its nearness to the low discharges. Almost all of the error indicators are under one such as shown in Figure 5.

Table 2. Optimum Parameters and Performance Indicators

Parameter	Terauchi	Ciriung
A_0	0.285	0.028
A_1	0.05	0.001
Ha_1	5	5.893
A_2	0.061	0.022
Ha_2	50.741	60
B_0	0.216	0.001
B_1	0.471	0.005
Hb_1	30	30
C_0	0.051	0.004
C_1	0.293	0.006
Hc_1	60	0.003
D_1	0.001	0.001
Ha_0	9.1	375.073
Hb_0	23.686	27.035
Hc_0	41.13	307.414
Hd_0	502.966	13.5
Ha	5.613	197.271
Hb	7.731	344.628
Hc	28.419	27.157
Hd	882.181	117.888
R	0.943	0.909
RMSE	3.067	1.018
MAE	1.288	0.819
LOG	0.244	0.262
χ	0.574	0.706
X^2	0.561	0.841
RE	0.362	0.714
RR	0.422	1.083

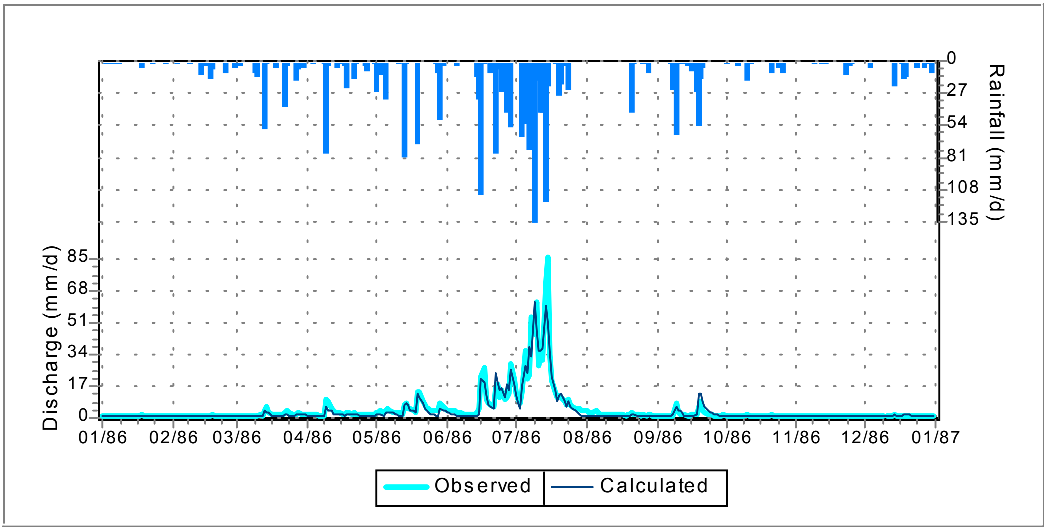

Figure 2. Hydrograph for Terauchi Watershed in 1996

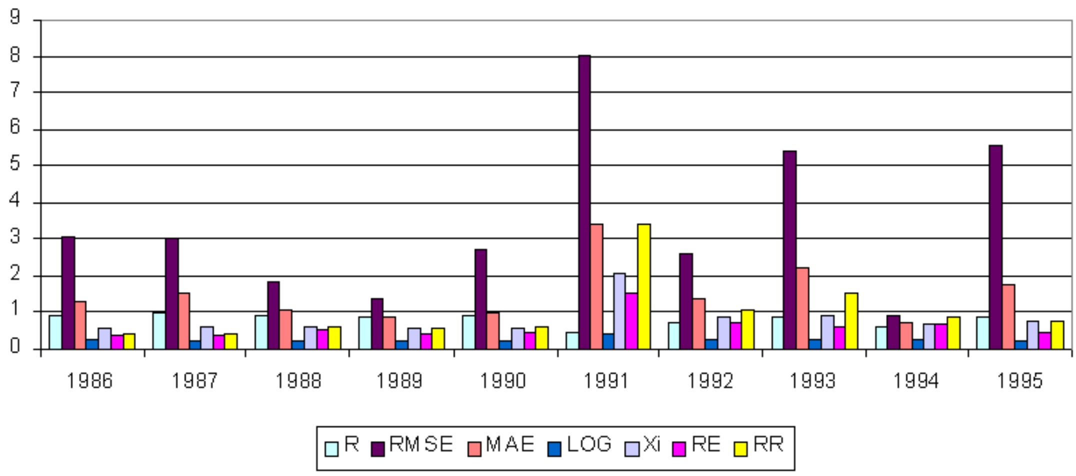

Figure 3 Correlation and Errors between measured and calculated for Terauchi Watershed (1986~1995)

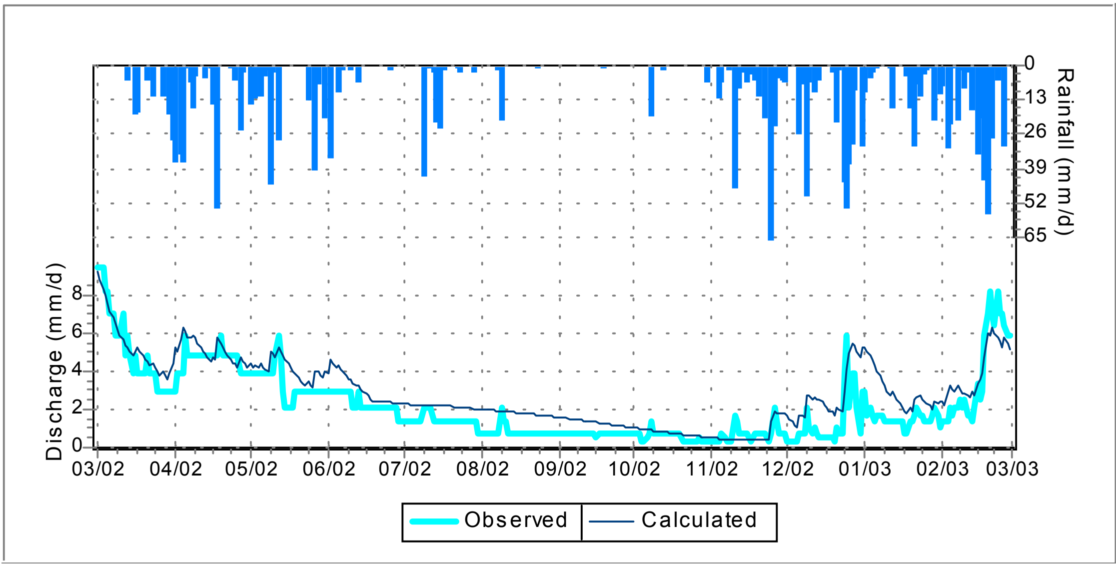

Figure 4. Hydrograph for Ciriung Watershed in 2002/2003

Figure 5. Correlation and Errors between measured and calculated for Ciriung Watershed in 2002/2003

This optimization process ran very fast in coming to convergence, i.e., it took time only in seconds. The number of iteration was seldom exceeding 100 times. It is possible in this process to set one or more parameters with known values to be constant. Eye fitting was also possibly done in the process in order to make the calculation more approaching to peak discharges or on the other way. This could be successfully done however with the compensation on decreasing the coefficient of correlation and effecting error indicators.

This optimization technique has been made in the form of application package program[2] in Window environment, using Delphi programming language (Appendix 2). Basically this program receives inputs of date, rainfall, potential evapotranspiration, and river discharges in mm/day unit. The outputs can be saved in the form of ASCII file for all parameters, water balance, intercept, slope, coefficient of correlation, error indicator including all water flow components (*Y, Ya, Yb, Yc dan Yd*). The hydrographic and regression curves can be saved in JPG file. The initial value of D_I could be given directly after data loading, and the process of **CHECK** would result in the initial values for water level at each reservoir. If the initial values had given reasonable water balance, then **OPTIMIZE** could be run in order to update parameters. Subsequently, the optimization process could be started and optimum parameter could be immediately obtained. Although this application is still limited for Standard Tank Model, yet this technique could be easily developed further to accommodate other type of Tank Model.

Conclusions

This paper has discussed parameter optimization technique for Standard Tank Model and here was tested for 2 (two) watersheds in Indonesia and Japan. This optimization technique is also equipped with the procedure for estimating the initial water level in each reservoir before starting the optimization process. The optimization process uses Marquardt algorithm showing its effectiveness and speed to determine the parameters. The test results of the two watersheds showed that the Standard Tank Model could represent the relation between rainfall minus potential evapotranspiration and water debit, which resulted in very good performance in terms of water balance, coefficient of correlation, and error indicators. This optimization technique has been made available in the form of application package program and readily developed for other Tank Models with different structures.

Acknowledgement

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[2] The application program will be delivered free of charge upon request to the first author.

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Appendix 1. Algorithm for Standard Tank Model in Pascal Language

```
procedure TankModel(B:ArrayM; Xi:single; var Yx:single);
var
  YA0,YA1,YA2,YB0,YB1,YC0,YC1,YD1: single;
procedure TankA(Ha:real; B:ArrayM; var YA0,YA1,YA2:single);
begin
  YA0:=B[0]*Ha;
  YA1:=B[1]*(Ha-B[2]);  if YA1<0 then YA1:=0;
  YA2:=B[3]*(Ha-B[4]);  if YA2<0 then YA2:=0;
end;{TankA}

procedure TankB(Hb:real; B:ArrayM; var YB0,YB1:single);
begin
  YB0:=B[5]*Hb;
  YB1:=B[6]*(Hb-B[7]);  if YB1<0 then YB1:=0;
end;{TankB}

procedure TankC(Hc:real; B:ArrayM; var YC0,YC1:single);
begin
  YC0:=B[8]*Hc;
  YC1:=B[9]*(Hc-B[10]); if YC1<0 then YC1:=0;
end;{TankC}

procedure TankD(Hd:real; B:ArrayM; var YD1:single);
begin
  YD1:=B[11]*Hd;
end;{TankD}

begin {Main for TankModel}
  Ha:=Ha+Xi;
  TankA(Ha,B,YA0,YA1,YA2);
  Ha:=Ha-(YA0+YA1+YA2);

  Hb:=Hb+YA0;
  TankB(Hb,B,YB0,YB1);
  Hb:=Hb-(YB0+YB1);

  Hc:=Hc+YB0;
  TankC(Hc,B,YC0,YC1);
  Hc:=Hc-(YC0+YC1);

  Hd:=Hd+YC0;
  TankD(Hd,B,YD1);
  Hd:=Hd-YD1;
```

```
  Yx:=YA1+YA2+YB1+YC1+YD1;
end;{End of TankModel}
```

Appendix 2. Application Program for Optimization of Tank Model's Parameters