MATHEMATICAL MODEL FOR PADDY DRYING[1]

(Model Matematika Untuk Pengeringan Gabah)

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ABSTRAK

Tujuan utama dari penelitian ini adalah mempelajari karakteristik pengeringan dan membuat model pengeringan lapisan tipis dari gabah ramping berbentuk silindris. Karakteristik pengeringan sangat berguna untuk menentukan penampilan mesin pengering yang menggunakan berbagai sumber bahan bakar seperti sinar matahari, minyak tanah dan alternatif energi lainnya.

Penelitian ini di latar belakangi dan dikembangkan dari postulat dimana penampilan pengeringan gabah dalam bentuk tumpukan dapat diduga dari rata-rata perilaku individu biji. Individu gabah dianggap sebagai benda silindris dimana pola migrasi uap akan mengikuti pola diffusi dalam unit silinder tak terhingga.

Berdasarkan postulat diatas telah dikembangkan model pengeringan lapisan tipis silindris tak terhingga. Hasil yang diperoleh adalah model silindris telah mampu menghitung perubahan kadar air dari pengeringan lapisan tipis dengan sangat teliti. Dari model ini juga didapatkan koefisien pengeringan K dan kadar air keseimbangan dinamis Me untuk varietas IR-36. Model bola dapat juga dipakai untuk menduga perubahan kadar air dari gabah ramping, tetapi menghasilkan nilai k dan Me yang berbeda dengan hasil perhitungan dari model silindris.

INTRODUCTION

Big harvest usually occures in the rainy season. During the rainy season lack of sunlight intensity creates problems in rough rice drying. Many attemps have been made to dry rough rice with artifical dryers, mainly since 1969, but the results fell short of expectations because many dryers were idle or used inefficiently (Directorate of Food Crop Economics, 1979). Previously many researchers had attempted to conduct research on rough rice drying. In Indonesia, however, there were no published report on basic research to study the drying mechanism.

In grain drying, the controlling process occurs within the falling period. Previously, many drying models were based on the infinite slab and the spherical form for short or medium grain (Henderson and Pabis, 1961; Nishiyama, 1982;

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[1] Continuation from previous research on Drying of Long Grain, presented at the International Drying Symposium 84 in Kyoto, July 1984.

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Pabis and Henderson, 1961; Steffe and Singh, 1980). Since the varieties of paddy in Indonesia belongs to the indica group which has a slender body, it is logical to develop a cylindrical drying model, for better representing the drying mechanism.

The main objectives of this research were first to clarify the drying characteristics of the tropical long or slender grains, to develop thin layer drying model based on diffusivity of vapour in unit cylinder and to develop deep layer drying model.

THEORITICAL CONSIDERATION

Cylindrical-Thin-Layer-Drying Model

If one kernel of rough rice is assumed as a cylindrical body, the mass or moisture balance within the kernel can be expressed in term of the following partial differential equation (Bird, Steward and Lightfoot, 1960; Brooker, Bakker-Arkema and Hall, 1974)[1].

$$\frac{\partial w}{\partial t} = D\left\{ \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} \right\} \qquad (1)$$

Assuming that no differences in moisture content along the z-direction. For deep bed drying analysis, it is necessary to get the average moisture content of the kernels. Solving Equation (1), the average moisture content is given by (Brooker *et al.,* 1974).

$$\frac{\bar{w} - wE}{wi - wE} = 4 \sum_{n=1}^{\infty} 1/\lambda_n^2 \, E\,xp\,(X_1^2 \lambda_n^2 /4) \qquad (2)$$

Where X_n is the positive roots of the zeroth order Bessel function of the first kind and X_1 is dimensionless time define as $X_1 = (A_p/V_p)\sqrt{Dt}$.
To simplify X_1 the authors introduce new variable:

$$X = X_1^2 = (A_p/V_p)^2\,Dt = Kt \qquad (3)$$

K is called the drying constant and has a unit of $\min^{-1}$ and is a function of the shape and diffusifity of the kernel.

Nishiyama (1982) had succesfully developed an approximate equation of sphere drying model of rice. Using the similar procedure, a regression for $0 \leqslant X_1 \leqslant 10$ was developed to replace the infinite series in equation (2). For the same value of X_1, the new regression equation could estimate very closely the equation (2) for $n = 40$, with standard deviation of 0.000808. This regression equation is called approximate equation for cylindrical model. If parameter X_1 expressed in terms of parameter X as in equation (3), the approximate equation can be written as:

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[1] See Nomenclature on Page 37 for the explanation of the following equations.

$$S(X) = \begin{cases} 1 + 0.265907X^{1.0185} + 0.024801X^{2.41975} - 1.1275X^{0.5} & \text{for } 0 < X \leqslant 0.64 \\ 0.69154\, e^{(-1.445766X)} + 0.01346633\, e^{(-7.617876X)} & \text{for } X > 0.64 \end{cases} \tag{4}$$

This equation is called the cylindrical-thin-layer drying model (CTLD model).

Application to Deep-Bed Drying

In the followings, an analysis is developed in attempt to apply the cylindrical model to packed grain divided into N_n layers. The moisture content and the thermal properties of the drying air were calculated at each layer. Output from the first layer become the input for the next layer.

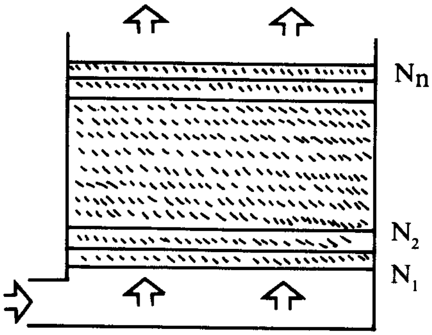

Fig. 1. Packed bed drying consist of N_n layers.

Temperature, relative humidity and moisture content changes were calculated using the basic drying theory and thermodynamics relationship. Grain temperature was calculated with the assumption that heat accumulated was equal to the heat convected from drying air minus heat of evaporation. This was given by:

$$\frac{d\theta p}{d t} = \frac{-\alpha A(\theta p - \theta a) - (\Delta h\, m_d\, dw/dt)/100}{m_p\, cp_p} \tag{5}$$

where m_d is the dry matter of grain, subcript p and a for rough rice and drying air respectively. Air temperature was calculated on the assumption that heat released by the drying air was used for heating rough rice and evaporation, with zero heat losses. The equation was expressed as:

$$d\theta_a = \frac{m_p\, cp_p\, (d\theta_p/dt) + (dw/dt)\, m_d\, \Delta h/100}{u\, \rho\, a\, cp_a} \tag{6}$$

Rate of drying is calculated after taking the differential of equation (4) with respect to time which gives:

$$\frac{dw}{d\theta} = \begin{cases} K(M)(0.270826X^{1.0185} + 0.060012X^{1.4975} - 0.56375X^{-0.5} & \text{for } (0 \leqslant X \leqslant 0.64) \\ K(M)(-0.999815e^{(-1.445766X)} - 0.102584e^{(-7.617876X)} & \text{for } (X > 0.64) \end{cases} \quad (7)$$

where $M = (w_i - w_E)$

Thermal properties of the grain were expressed as:

Specific heat (Morita and Singh, 1977): $Cp_p = 4186.8\,(0.2201 + 0.01301w'$ (8)

Latent heat of evaporation (Nishiyama, 1980):

$$h = (2500 - 2.34\theta_p)(.68Exp(-0.114w) + 1)/1000 \quad (9)$$

Thermal and physical properties of moist air are expressed by the following equation:

Specific heat (Nishiyama, 1980): $Cp_a = 1.005 + 1.85\,x$

Local convective heat transfer coefficient (Bird *et al.*, 1960): (10)

$$jH = \frac{\alpha\,(Pr)^{2/3}}{Cp_a\ \rho\ a^u} = m(Re)^n$$

where $Re = G/Ab\ \mu\ \psi\,)$

For $Re < 50$, $m = 0.91$, $n = (-0.51)$ and for $Re > 50$, $m = 0.61$, $n = (-0.41)$. ψ depend on the shape of the body, for cylindrical body $\psi = 0.91$. jH is known as Chilton Colburn factor.

Mass density of dry air (ideal gas) (Hara and Nishiyama, 1983):

$$\rho_{da} = 0.00348\,(\frac{Pt}{T}) \quad (12)$$

Mass density of moist air:

(Hara & Nishiyama, 1983) $\rho\ m = \rho\ da\,(1\text{-}0.622pv/pt)$ (13)

Vapour pressure (Hara & Nishiyama, 1983): $pv = x/(0.622 + x)$ (14)

After passing one layer the humidity ratio x is increased as:

$$x_o = x_i + \frac{m_d\, dw/dt}{100\,u\,\rho_{da}} \quad (15)$$

EXPERIMENTAL PROCEDURES

Apparatus in Fig. 2 was used for measuring parameters K and w_E from equation (4). This apparatus was designed and constructed in such a away to enable the authors to produce a certain level of temperature and relative humidity of the air (Terui, Kimura and Nishiyama, 1979).

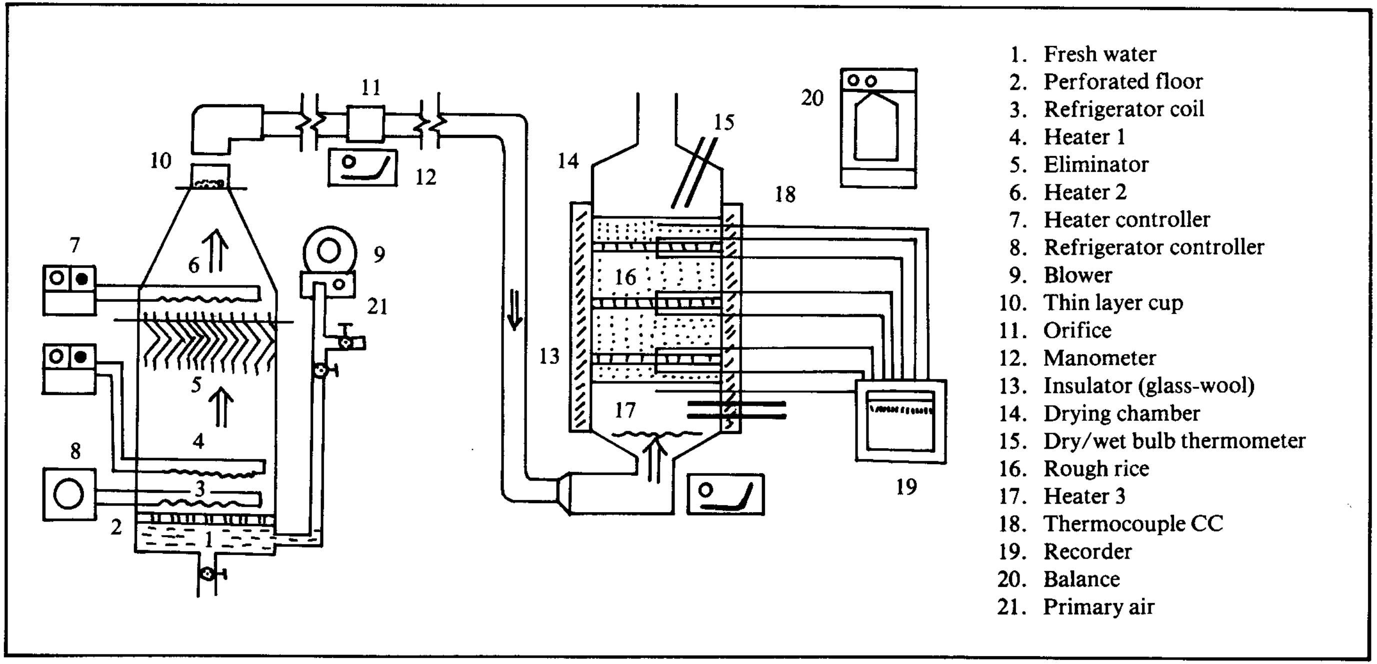

Fig. 2. Schematic of drying experimental apparatus.

Ambient air was passed through a water bath maintained at a certain temperature and then heated to produce constant humidity ratio by heater 2 to get a certain value of relative humidity and temperature. For thin layer experiment, thin layer cup was placed on the top of the drying chamber (part A in Fig. 2) filled with a two-grain-thickness layer of rough rice. Air flow rate at 0.1 m/s was then passed through this layer.

For deep bed drying experiment, thin layer cup was replaced by a PVC drying chamber. Air flow was measured by orifice. Auxilliary heater 3 was installed to overcome extra load in maintaining constant temperature. The packed bin has 30 cm height and 25 cm inside diameter with approximate loading capacity of 9 kg wet rough rice. The bin wall was insulated with 5 cm thick glass wool and could be dissambled during weighing process.

Rough rice sample (IR-36) was obtained from Bogor Research Institute for Food Crops. Experimental run was designed for five temperature levels at 35, 40, 45, 50 and 55°C for thin layer drying and 45°C for deep bed drying.

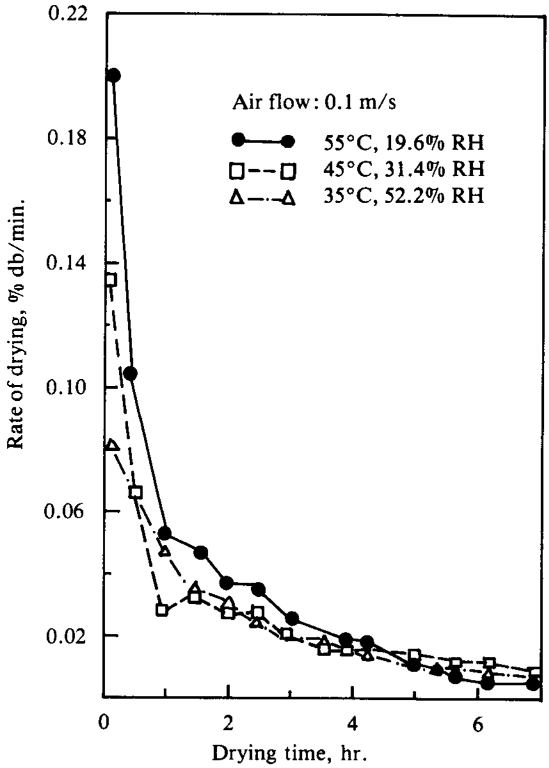

Fig. 3. Drying time vs rate of drying for thin layered rough rice.

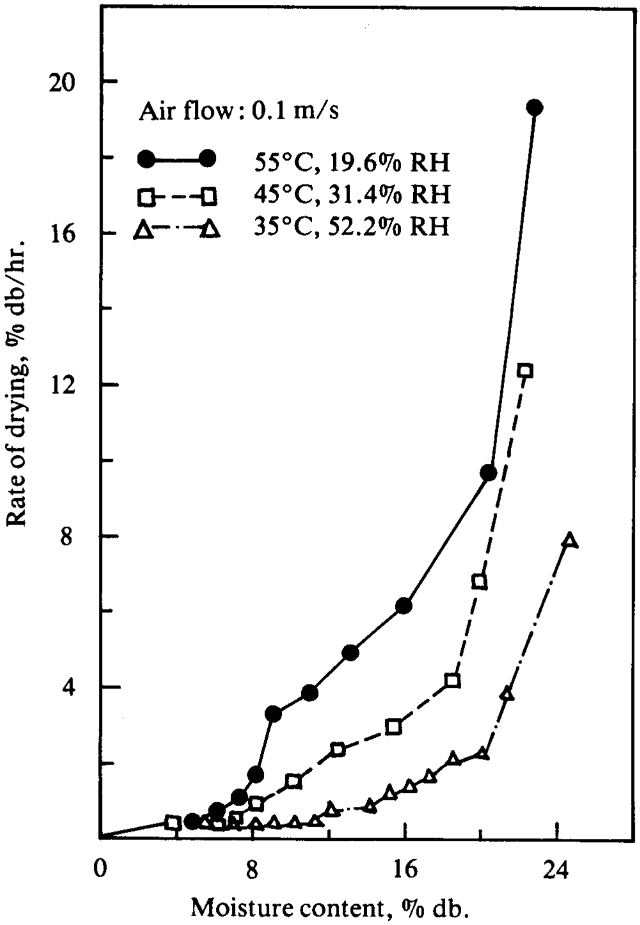

Fig. 4. Moisture content vs rate of drying for thin layered rough rice.

RESULTS AND DISCUSSIONS

Thin-Layer

Figs. 3 and 4 are typical experimental data showing the rate of drying for thin layered rough rice under constant air flow rate of 0.1 m/s and three levels of RH 19.6%, 31.4% and 52.2%. The figure also indicate that the drying process could be divided into basically two distinctive regions within the observed falling rate period. For these three temperature levels under study, the higher drying rate continued to persist untill the moisture content reached a value of 14% wb. Higher drying temperature tended to increase the lower limit of moisture content for high drying rate.

Deep Bed

Moisture content

Rate of change of moisture content under deep bed drying condition is shown in Fig. 5 in terms of drying time. All the data were taken under constant drying temperature of 45°C and 38.7% RH. The top, middle and bottom layer conditions were respectively taken at a depth of 4-6, 14-16 and 24-26 cm from the surface of the bulk in the drying chamber. At the top and middle layer, the moisture content increased initially before decreasing gradually toward the equillibrium moisture content of that particular layer.

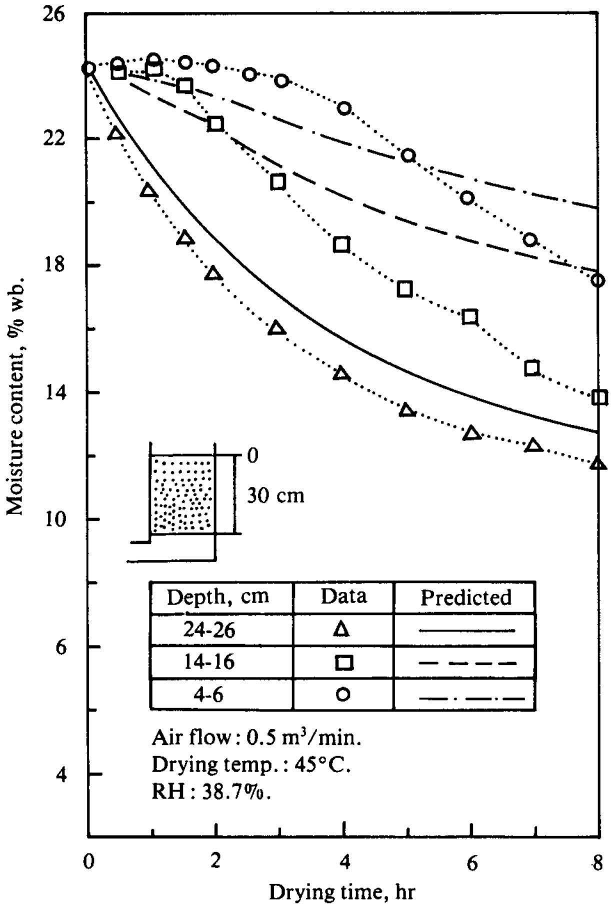

Fig. 5. Predicted moisture content vs experimental data for deep bed drying.

Temperature

Fig. 6 show how the void (air) temperature changed at various layers within the drying chamber. The temperature at the bottom and middle layer showed a similar pattern of increment whereas the temperature at the top layer showed a region where relatively constant temperature level exist between the first and third hour of drying time.

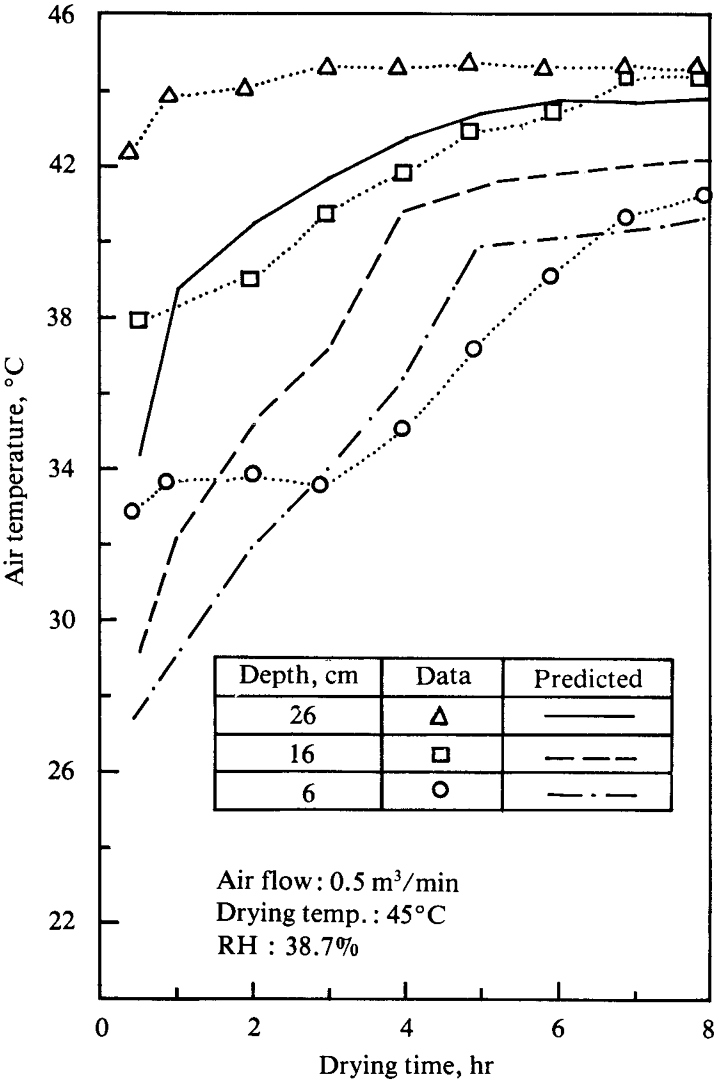

Fig. 6. Predicted temperature change vs experimental data for deep bed drying.

Drying Coefficient K

In order to compute the moisture change over time using equation (2) or its approximate form in equation (4), one has to know the value of the drying constant K. In this study, the determination of this coefficient was done following the procedure developed by Nishiyama (1982). For cylindrical-thin-layer-drying model (CTLD) the K value for IR-36 variety was found to be: form:

$$K_{cyl} = \mathrm{Exp}(8.21589 - 4444.89/T) \tag{16}$$

where T is the absolute temperature (°K). The corresponding K value for sphere model was also determined which has the following form:

$$K_{sphere} = \mathrm{Exp}(9.72234 - 4858.94/T) \tag{17}$$

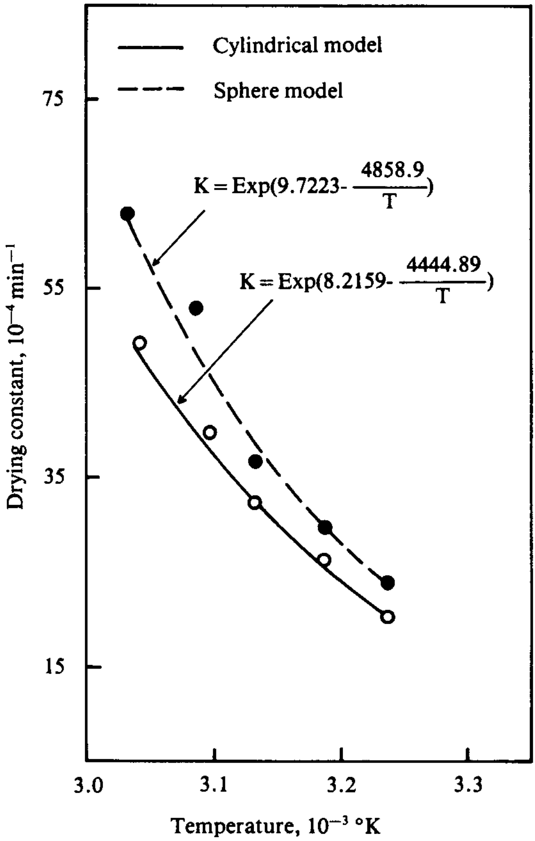

Fig. 7. Change of value of drying constant of rough rice with respect to temperature.

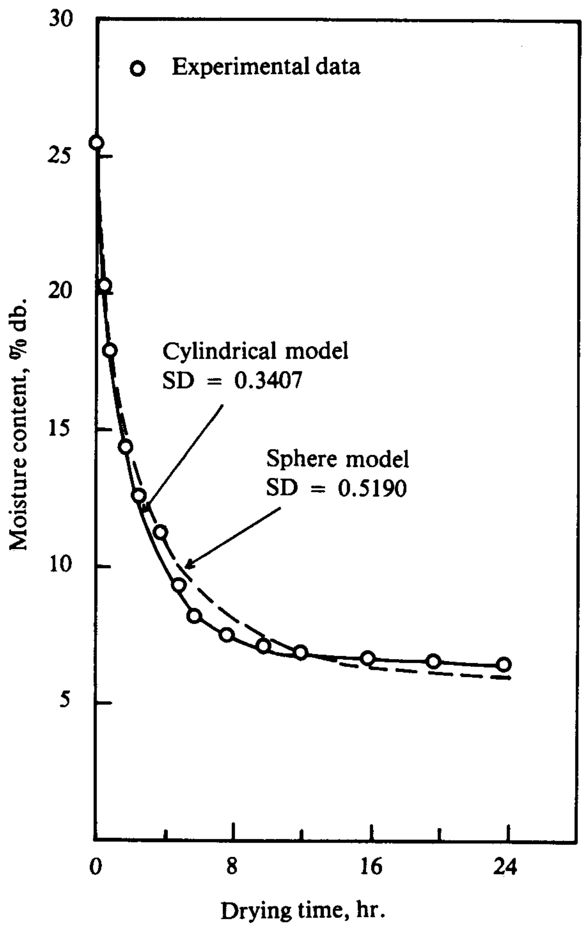

Fig. 8. Predicted change of moisture content using cylindrical model for thin layered rough rice.

Nishiyama (1982) had found that for the japonica variety, the equation for K:

$$K_{sphere} = Exp(8.350\text{-}4449/T) \quad (18)$$

Which shows a slight difference in the value of K_{sphere} from IR-36.

Fig. 7 shows both value of the drying constant (equations 16 and 17), based on the cylindrical and sphere models, as the function of the drying temperature.

Results of computation with the models are shown in Fig. 8 for thin-layer drying with drying temperature of 45°C. The figure shows that both models were in good agreement which the experimental data, in which the CTLD model seemed more superior in comparison to the sperical model.

From the CTLD model, the authors also had derived an equation for the dynamic equillibrium moisture content expressed in terms of percent-dry basis.

$$w_E = 24.7123 - 1.73384(\Delta\theta') + 0.03849(\Delta\theta')^2 \qquad (19)$$

$\Delta\theta'$ is the differences between the dry and wet bulb temperatures. Equation (19) has been tested and found to be valid within the limited range of 19% to 52% RH.

When this and other partinent relationship mentioned above were substituted into equation (4) through equation (19), the predicted moisture and temperature change were as shown in Fig. 5 and 6.

For the case of moisture content change in the grain, the model seemed to underestimate the experimental data at the early drying process particularly at the top layer and overestimated the data at the bottom and middle layer. One reason for this discrepancy might be due to the limited validity of equation (19) for the condition of high RH as for the case at the middle and top layer where moisture content of the air reached saturated condition. Another limitation of the model was that it could be applied for the case where the dimensionless time exeeds a value of three.

Fig. 6 shows the predicted and measured value of drying air temperature change when passed through the bulk of the grain. The lack of precision, so far, was influenced by the degree of accuracy in predicting the moisture content of the grains, the estimation of the local convective heat transfer coefficient of the packed bed and the effective surface area of the grain which undergoing the heat and the moisture exchange.

CONCLUSIONS

1. The drying characteristics of rice grain indicated that up to 28% db of initial moisture content, the occurrence of constant drying rate period was not observed.
2. Moisture migration within the long grain could be explained through diffusion theory within a finite cylindrical body as shown in figs 7 and 8 (CTLD model).
3. The drying model based on vapour diffusion in a cylindrical body was found superior for calculating moisture change in thin layer drying of long grain.
4. Under a certain range of precision, the sphere model could also be applied to the case of long grain.

SUGESTIONS

In order to improve the current model, some considerations should be made: a) Inclusions of moisture saturated drying front, and b) Accurate prediction of the effective grain surface area for heat and moisture exchange.

Nomenclature

A = surface area, m^2
Cp = specific heat, J/(kg.°K)
D = coefficient of diffusivity, m^2/min or m^2/s
G = superficial velocity, kg/(m^2s)
Δh = latent heat of evaporation, J/kg
jH = the Chilton-Colburn factor, (-)
K = drying constant, 1/min
M = moisture content differences (w_i-w_e), % db
m = mass, kg
n,m = constants, (-)
p = pressure, Pa
Pr = Prandtl number, (-)
r = radial, m
Re = Reynold number, (-)
t = time, s or min
T = drying temperature, °K
u = volumetric velocity, m^3/s
V = volume, m^3
w = moisture content, % db
X = dimensionless time as the function K.t
X_1 = dimensionless time in equation /2/
x = absolute humidity, kg water/kg dry air

Greek symbol

α = local convective heat transfer coefficient, W/(m^2°K)
θ = temperature, °C
$\Delta\ \theta'$ = temperature differences between dry and wet bulb, °C
λ = positive root of the zeroth order Bessel function of the first kind, (-)
μ = viscosity, kg/(m.det)
ρ = mass density, kg/m^3
ψ = constant, (-)

Subscript

a = air
b = per volume of bed
d = dry matter
da = dry air
E = state of equilibrium
i = initial
m = moist air
o = output

p = grain
t = total
v = vapour

REFERENCES

Bird, R.B., W.E. Stewart and E.N. Lightfoot. 1960. Transport phenomena. John Wiley & Sons, Inc., London.

Brooker, D.B., F.W. Bakker-Arkema and C.W. Hall. 1974. Drying cereal grains. AVI Publ. Co., Westport, Connecticut.

Directorate of Food Crop Economics. 1979. Survey on dryers. Paper presented in: Post-Harvest Workshop, Jakarta, January 16-18.

Hara, M. and Y. Nishiyama. 1983. Mathematical expression of moist air conditions. J. of The Faculty of Agriculture, Iwate University. 16: 107-121.

Henderson, S.M. and S. Pabis. 1961. Grain drying theory I. Temperature effect on drying coefficient. J. Agric. Engineering Res. 6: 169-174.

Morita, T. and R. P. Singh. 1977. Physical and thermal properties of short-grain rough rice. ASAE paper no; 77-3570. St. Joseph, MI.

Nishiyama, Y. 1982. Deep layered grain drying simulation by the sphere drying model. J. of The Faculty of Agriculture, Iwate University. 16: 45-59.

Pabis, S. and S.M. Henderson. 1961. Grain drying theory III. The air/grain temperature relationship. J. Agric. Engineering Res. 7: 21-26.

Steffe, J.F. and R.P. Singh. 1980. Parameters required in the analysis of rough rice drying. Drying' 80, Proc. of The Second IDS. 2: 256-262.

Terui, T., T. Kimura and Y. Nishiyama. 1979. A conception on determination of equilibrium moisture content under drying process of cereal grain. J. of The Soc. of Agric. Machinery. 2: 271-277.