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Connectivity Problem of Wildlife Conservation in Sumatra: A Graph Theory Application

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ABSTRACT

In this paper, the problem of connectivity of patchy conservation sites was approached by the use of graph theory. Determination of the so-called core sites was subsequently conducted by establishing the cover areas formulated in the framework of integer linear programming, connecting the unconnected cover areas by application of Dijkstra algorithm and heuristically pruning the unused covers to secure the minimum connected cover areas. An illustrative example of this method describes the wildlife conservation in Sumatra. The connectivity problem of 20 *districts in* 3 *provinces of Jambi, Riau and West Sumatra inhabited by* 10 *species of wildlife were considered.*

Keywords: Connectivity problem, covering-connecting-pruning, Dijkstra algorithm, wildlife conservation, Sumatra.

2010 Mathematics Subject Classification: 05C90, 92D40.

1 Introduction

Connectivity of networks is one of the most fundamental and useful notions for analyzing various types of network problems and now is phrased in metapopulation theory. Connectivity in conservation biology can be defined as the degree to which organisms can move through the landscape, commonly measured either by dispersal rate or dispersal probability. In other words, connectivity can be viewed as a problem of connecting territories of great biological importance, i.e., corridors creation problem. At intermediate time scales connectivity affects migration and persistence of metapopulations (Ferreras, 2001) and at the largest time scales it influences the ability of species to expand or alter their range in response to climate change (Opdam and Wascher, 2004). Habitat connectivity is especially important when habitat is rare, fragmented, or otherwise widely distributed and can be a critical component of reserve design (Minor and Urban, 2008).

Researches on the connectivity problem are some. (Gurrutxaga *et al.*, 2011) studied the connectivity of protected area networks based on key elements located in strategic positions within the landscape. Recent methodological developments, deriving from the probability of connectivity index, was applied to evaluate the role of both individual protected areas and links in the intermediate landscape matrix as providers of connectivity between the rest of the sites in the network. (Minor and Urban, 2008) used graph theory to characterize multiple aspects of landscape connectivity in a songbirds habitat network in the North Carolina Piedmont. (Pascual-Hortal and Saura, 2008) systematically compared a set of ten graph-based connectivity indices, evaluating their reaction to different types of change that can occur in the landscape and their effectiveness and proposed a new index that achieves all the properties of an ideal index. Mathematical formulations and solution techniques for a variant of the connected subgraph problem in wildlife conservation subject to a budget constraint on the total cost was investigated by (Dilkina and Gomes, 2010). Several mixed-integer formulations for enforcing the subgraph connectivity requirement were proposed. A paper by (Nagamochi, 2004) surveys the recent progress on the graph algorithms for solving network connectivity problems. One discussed the optimization strategy of network connection is (Lu, 2013). A review on recent applications of graph model and network theory to habitat patches in landscape mosaics is provided by (Urban *et al.*, 2009). A book that focuses on practical approaches, concepts, and tools to model and conserve wildlife in large landscapes is (Millspaugh and Thompson, 2009).

The current paper describes the application of graph theory in connectivity problem of wildlife conservation. We consider the problem of determining the minimum connected core sites in Sumatra, where 10 species of wildlife inhabited in 20 districts in Jambi, Riau and West Sumatra provinces are considered. We shall follow the approach developed by (Cerdeira *et al.*, 2005), where the minimum connected cover areas were determined by using integer linear programming, Dijkstra algorithm and heuristic pruning method.

2 Graph Model of Connectivity Problem

2.1 Notation

The following notation can be found in most standard textbooks on graph theory and network flows, e.g., (Ahuja *et al.*, 1993; Diestel, 2000). *Graph* G is defined as an ordered pair of sets (V, E), where V is set of *nodes* and E is a set of 2-element of V, called *edges*. A graph has *order* n if its number of nodes is n, written by $|G| = n$. Graphs are *finite* or *infinite* according to their order. A node v is *incident* with an edge e if $v \in e$. Two nodes u, v of G are *adjacent*, or *neighbors*, if (u, v) is an edge of G. It is denoted by $A(v)$ the set of all adjacent nodes of v. *Directed graph* is an ordered pair of sets $(V, \tilde{E})$, where V is set of nodes and $\tilde{E}$ is a set of ordered pairs of two nodes. A graph $G' = (V', E')$ is said to be a subgraph of graph $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. *Multigraph* is a graph where multiple edges between the two nodes are allowed, e.g., set of edges E becomes a multiset. *Weighted graph* is a graph where each edge is bound with a certain value, called *weights*. Those weights can be costs, profits, lengths of the edges etc. A *path* in a graph is a sequence of distinct vertices $(v_1, v_2, \ldots, v_n)$, such that (v_{i-1}, v_i) is an edge, for all $i = 2, \ldots, n$. The *length* of a path is the number of edges in the path. A *shortest path* is a path of minimum length. A graph is *connected* if there is a path from each node to any other node in the graph.

Let suppose $V := \{1, 2, \ldots, n\}$ and $C := \{C_1, C_2, \ldots, C_k\}$, where $C_i \subseteq V$ for $i \in I := \{1, 2, \ldots, k\}$. Sets C_i, where $i \in I^* \subseteq I$, are *covers* of set V if they satisfy

$$\bigcup_{i \in I^*} C_i = V. \tag{2.1}$$

If, for instance, $V = \{a, b, c, d, e, f, g\}$ and $C = \{C_1, \ldots, C_5\}$, where $C_1 = \{a, b, g\}$, $C_2 = \{a, c, g\}$, $C_3 = \{b, e, g\}$, $C_4 = \{d, e, f\}$ and $C_5 = \{a, c, f\}$, then $\{C_1, C_4, C_5\}$ is one of covers of V since $C_1 \cup C_4 \cup C_5 = V$ as required by (2.1). Suppose that a weight $w_i > 0$ is attached to each C_i. The *set covering problem* is a problem of determining covers with minimum weight, and can be formulated as a linear programming:

$$\min \sum_{i=1}^{k} w_i x_i \quad \text{s.t.} \quad \sum_{i=1}^{k} a_{ij} x_i \geq 1, \quad j = 1, \ldots, n, \tag{2.2}$$

where $x_i = 1$ if C_i is a cover of V for $i = 1, \ldots, k$, otherwise $x_i = 0$ and $a_{ij} = 1$ if $i \in C_i$, otherwise $a_{ij} = 0$. Here n denotes the number of elements in V. If we set $w_i = 1\,(i = 1, \ldots, k)$, the minimum covering set for V of previous example is $\{C_3, C_4, C_5\}$ with minimum weight 3.

Connectivity problem of wildlife conservation, which involves a number of conservation sites and wildlife species, can be loosely defined as a problem of selecting a fewest number of connected sites that cover all species.

2.2 Graph Model

Consider a wildlife conservation area which consists of n sites. In graph theory notation, these sites are denoted by nodes and any two connected nodes are denoted by two adjacent nodes, where road connecting them is an edge. Suppose that there are m species inhabited in the area. Based on the work of (Cerdeira *et al.*, 2005), connectivity problem in the area is solved in three steps: covering, connecting and pruning.

2.2.1 Covering

For $i = 1, \ldots, n$, define the following binary decision variables:

$$x_i = \begin{cases} 1 & ; \quad \text{if sites } i \text{ is selected} \\ 0 & ; \quad \text{if otherwise.} \end{cases} \tag{2.3}$$

The covering problem is then aimed to minimize the number of selected sites:

$$\min \sum_{i=1}^{n} w_i x_i \quad \text{s.t.} \quad \sum_{i \in K_j} x_i \geq 1, \quad j = 1, \ldots, m, \tag{2.4}$$

where constraints in (2.4) ensure that each species populates at least in one selected site. Here K_j $(j = 1, \ldots, m)$ denotes the set of sites where species j inhabited. If the optimal covering set is connected then the process is stopped, otherwise we proceed to the second step for connecting.

2.2.2 Connecting

Connecting of unconnected covers is undertaken by transforming the representing unconnected graph into connected one. In this stage, the shortest path of the respecting unconnected graph should be sought by Dijkstra algorithm (Dijkstra, 1959). For a given source node v_s in the graph, the algorithm finds the path with lowest cost, i.e., the shortest path, between that nodes and every other node v_i. The algorithm divides the nodes into permanently labeled and temporarily labeled nodes. The distance label to any permanent node represents the shortest distance from the source to that node. For any temporary node, the distance label is an upper bound on the shortest path distance to that node. The basic idea of the algorithm is to fan out from node v_i and permanently label nodes in the order of their distances from node v_s. Dijkstra's algorithm will assign some initial (total) distance label d and will try to improve them step by step (Ahuja *et al.*, 1993):

1. distance label of source node: $d(v_s) = 0$.
2. for all $v_j \in V - \{v_s\}$ do $d(v_j) = \infty$.
3. initial set of visited node: $S = \emptyset$.
4. initial set of unvisited node: $Q = V$.
5. while $Q \neq \emptyset$ do $v^* = \min_{v_i \in Q} d(v_i)$.
6. $S = S \cup \{v^*\}$ and $Q = Q - \{v^*\}$.
7. for all $v_i \in A(v^*)$ do if $d(v_i) > d(v^*) + \omega(v^*, v_i)$ then $d(v_i) = d(v^*) + \omega(v^*, v_i)$.
8. return.

Steps 1 and 2 assign to permanent label 0 for source node v_s and each other node v_j to temporary label ∞. Steps 3 and 4 initially create an empty set of visited nodes S and that of unvisited nodes Q consisting of all the nodes. While Q is not empty, Step 5 selects an element v^* of Q with minimum distance. Step 6 updates S and Q with respect to v^*. By Step 7 we calculate the distance label for each node v_i which is adjacent to v^* by considering its weight $\omega(v^*, v_i)$. If a new shortest path is found, we replace $d(v^*)$ with the new one. If the destination node has been marked visited or if the smallest tentative distance among the nodes in Q is ∞, then stop. The algorithm has finished. Select the unvisited node that is marked with the smallest tentative distance, and set it as the new v^* then go back to Step 6.

2.2.3 Pruning

In this step, Q should be pruned to become a minimal connected cover by removing all unused nodes. The procedure consists of two step (Cerdeira *et al.*, 2005):

1. This step comprises of three processes:
   (a) Identify neighborhoods of all nodes in Q.

Table 1: Conservation sites.

Province	Site or district
Jambi	(1) Kerinci, (2) Merangin, (3) Sarolangun, (4) Batanghari, (5) Muarojambi, (6) Tanjungjabung Timur, (7) Tanjungjabung Barat, (8) Bungo, (9) Tebo
Riau	(10) Kampar, (11) Kuantan Singingi, (12) Indragiri Hilir, (13) Indragiri Hulu, (14) Pelalawan
West Sumatra	(15) Pesisir Selatan, (16) Solok, (17) Sawahlunto, (18) Agam, (19) Tanah Datar, (20) Padang Pariaman

(b) Pick a node v_i in Q and mark all nodes adjacent to v_i.

(c) Temporarily remove v_i and inspect remaining nodes in Q. If the set of remaining nodes is a cover and connected graph, then permanently remove v_i and return to Step 1(b). Otherwise, proceed to Step 2.

2. In this step we inspect nodes of Q that remain unchecked by Step 1 and then proceed the pruning procedure.

   (a) Select an unchecked node in Q and mark all adjacent nodes.

   (b) Temporarily remove one marked node v_i and inspect remaining nodes in Q. If the set of remaining nodes is a cover and its graph is connected, then permanently remove v_i and return to Step 2(a). Otherwise, v_i cannot be removed and stop the process. Q is minimum connected cover.

3 Connectivity Problem of Wildlife Conservation in Sumatra

To illustrate our model we consider a connectivity problem of wildlife conservation in Sumatra, Indonesia. We consider a conservation area spanned by 20 sites or districts in 3 provinces as homes for 10 wildlife species (thus we have $n = 20$ and $m = 10$), where its map and graph representation are provided by Figure 1. The list of districts is given by Table 1 and that for species including their habitats are provided by Table 2. In the graph conservation sites are denoted by numbered nodes following Table 1. Weight between two adjacent nodes represents the distance between sites in kilometers.

To determine cover sites we solve optimization problem (2.4). Since there are 10 species considered then we have 10 constrains inside the problem. Constraint related to golden cat $(j = 5)$, for instance, can be expressed as follows:

$$\sum_{i \in K_5} x_i \geq 1 \;\Leftrightarrow\; x_1 + x_2 + x_{15} + x_{16} + x_{17} \geq 1.$$

Elements of K_j can be found in the last column of Table 2. With no relative importance among sites, i.e., $w_i = 1$ for all $i = 1, 2, \ldots, 20$, optimal solutions of the problem are $x_i = 1$ for $i \in \{2, 19\}$ and $x_i = 0$ otherwise, which mean that site 2 (Merangin) and site 19 (Tanah Datar) constitute as cover sites for the conservation area. It is easy to verify that all species can be found alive in these two sites. However these cover sites are unconnected and we should connect them

Table 2: Wildlife species and their habitats.

No	Local Name	International Name	Latin Name	Habitat (K)
1	Rangkong papan	hombill	*Buceros bicornis*	1,2,6,8,9,12,13,15–18
2	Harimau sumatra	sumatran tiger	*Panthera tigris*	1–8,11,13,15–20
3	Badak sumatra	sumatran rhinoceros	*Dicerorhinus sumatrensis*	1,2,5,7,15,16,17
4	Beruang madu	sun bear	*Ursus malayanus*	1–4,8,10,11,14–17,20
5	Kucing emas	golden cat	*Profelis aurata*	1,2,15,16,17
6	Siamang	sumatran gibbon	*Symphalangus syndactylus*	3,4,6,8–13,18,19
7	Kancil	mouse deer	*Tragulus kanchil*	5–14,19
8	Tapir	asian tapir	*Tapirus indicus*	1,2,5,7,10,11,13–18
9	Elang alap	hawk	*Accipiter trivirgatus*	1,2,15,16
10	Gajah sumatra	sumatran elephant	*Elephas maximus*	1,2,15,16,17

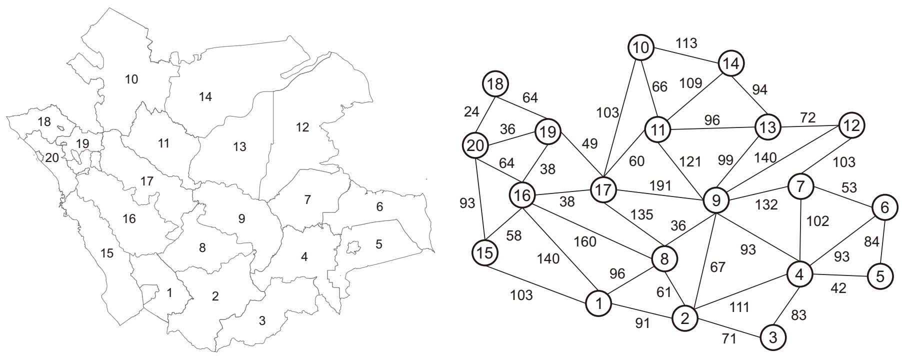

Figure 1: Conservation area and its corresponding graph.

first by seeking the shortest path in between. By applying Dijkstra algorithm we may find all the shortest paths from site 2 to other sites as depicted by Figure 2 (left). The shortest path between Merangin and Tanah Datar is shown by blackened path 2-8-17-19 with distance of 245 km. In the notion of Dijkstra algorithm we have $Q = \{2, 8, 17, 19\}$.

The last stage of the works is then to check whether the graph 2-8-17-19 provides a connected cover with minimum distance or not. Initial step of pruning is to identify the adjacency set of each node in Q and we have $A(2) = \{8\}$, $A(8) = \{2, 17\}$, $A(17) = \{8, 19\}$ and $A(19) = \{17\}$. If we temporarily remove node 2 from the list then we found that the set of remaining nodes $\{8, 17, 19\}$ is not a cover since hawk is not alive here. Thus node 2 is unremovable. In the second step of pruning, pick node 19 and temporarily remove it from the list. Since the set of remaining nodes $\{2, 8, 17\}$ forms a cover then node 19 can be permanently pruned. Over the new set $\{2, 8, 17\}$, by removing node 17 we discovered that $\{2, 8\}$ is a cover and node 17 can also be permanently pruned. The set of remaining nodes $\{2, 8\}$ therefore constitutes the connected cover with minimum distance, indicating that Merangin and Bungo, located 61 km apart, are conservation sites with the highes diversity, i.e., core sites, as all species reside in these areas.

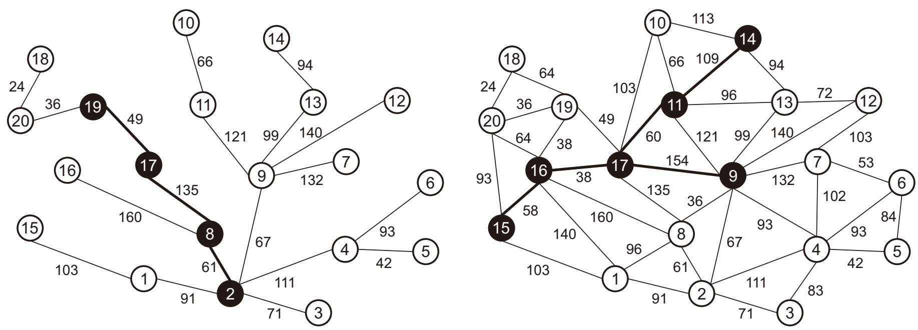

Figure 2: The shortest path and the Steiner tree.

4 Concluding Remarks

It has been shown that covering-connecting-pruning works well in determining core sites, i.e., connected cover sites with minimum distance, of wildlife conservation area. The effectiveness of this method is worthwhile when we consider complex cases where more sites and more species considered. Another interesting look relates to spatial consideration, where we may impose the possibility selected core sites located in at least two different provinces. Note that Merangin and Bungo obtained in previous problem are both located in Jambi province. To do this we may first modify weights w_i in (2.4) such that some sites are more important than others. For example, put $w_9 = w_{14} = w_{15} = 0$ and $w_i = 1$ otherwise. It means that Tebo (in Jambi province), Pelalawan (in Riau Province) and Pesisir Selatan (in West Sumatra province) have more importance to be considered as covers. Covering by integer linear programming (2.4) in fact provides these three sites as covers. Since there are more than two nodes to be connected, we now have the so-called Steiner tree problem, i.e., to find a tree of $G = (V, E)$ that spans $H \subseteq V$ with minimum total distance on its edges. In this case $H = \{9, 14, 15\}$. Figure 2 (right) depicts the Steiner tree of the problem, which can be obtained using algorithm developed by (Kou *et al.*, 1981; Nascimento *et al.*, 2012). However, pruning procedure gives 11-17-16 as the connected covers with minimum distance of 98 km along the tree. Now the core sites locate in two provinces: Kuantan Singingi in Riau; Solok and Sawahlunto in West Sumatra.

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