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The Organizing Committee is not responsible for any errors in the papers as these are individual author responsibility.

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FOREWORDS

President of the Indonesian Mathematical Society (IndoMS)

FOREWORDS

Assalamu'alaikum Warahmatullahi Wabarakatuh

Good morning and best wishes for all of us

It is my pleasure to say that the proceedings of the Second IndoMS International Conference on Mathematics and Its Applications (IICMA) 2013 from November 6 to November 7 at Yogyakarta-Indonesia finally published. The IICMA 2013 is the second IICMA, after IICMA 2009, which is organized by the Indonesian Mathematical Society (IndoMS) in collaboration with Department of Mathematics, Faculty of Mathematics and Natural Sciences, Gadjah Mada University and funded by Directorate of Research and Community Services, the Directorate General of Higher Education, Ministry of Education and CultureRepublic of Indonesia.

IICMA 2013 is one of the activities of IndoMS period 2012-2014. Organizing an IICMA 2013 is not only a continuing academic activity for IndoMS, but it is also a good opportunity for discussion, dissemination of the research result on mathematics including: Analysis, Applied Mathematics, Algebra, Theoretical Computer Science, Mathematics Education, Mathematics of Finance, Statistics and Probability, Graph and Combinatorics, also to promote IndoMS as a non-profit organization which has a member more than 1.400 people from around Indonesia area.

We would like to express our sincere gratitude to all of the Invited Speakers from the Netherlands, Georgia, India, Germany, Singapore and also Indonesia from Universities (ITB, UPI, University of Jember) and LAPAN Bandung, all of the speakers, members and staffs of the organizing committee of IICMA 2013. Special thanks to the Secretary of International Mathematics Union (IMU), the Directorate General of Higher Education, the Dean of Faculty of Mathematics and Natural Sciences-Gadjah Mada University, the Head of Department of Mathematics together with all staffs and students, also for supporting of lecturers and staffs as an organizing committee from Indonesian University, Padjadjaran University, University North of Sumatera, Sriwijaya University and Bina Nusantara University. Finally, we also would like to to give a big thanks for all reviewers who help us to review all papers which are submitted after IICMA.

With warmest regards,

Budi Nurani Ruchjana
President IndoMS 2012-2014

Chair of the Committee IICMA 2013

On behalf of the Organizing Committee of IndoMS International Conference on Mathematics and its Applications (IICMA) 2013, I would like to thanks all participants of the conference. This conference was organized by Indonesia Mathematical Society (IndoMS) and hosted by Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, Yogyakarta, Indonesia, during 6-7 November 2013.

In IICMA 2013, there will be 122 talks which consists of 10 invited and 112 contributed talks coming from diverse aspects of mathematics ranging from Analysis, Applied Mathematics, Algebra, Theoretical Computer Science, Mathematics Education, Mathematics of Finance, Statistics and Probability, Graph and Combinatorics. However, the number of paper which were sent and accepted in this proceedings is 33 papers. We would also like to give our

- Prof. Dr. S. Arumugam (Combinatorics, Kalasalingam University, India)
- Prof. Dr. Bas Edixhoven (Algebra, Universiteit Leiden-the Netherlands)
- Prof. Dr. Dr. h.c. mult. Martin Grotschel (Applied Math, Technische Universitat Berlin, Germany and International Mathematics Union)
- Prof. Dr. Kartlos Joseph Kachiashvili (Statistics, Tbilisi State University-Georgia)
- Prof. Dr. Berinderjeet Kaur (Mathematics Education, National Institute of Education, Singapore)
- Prof. Hendra Gunawan, Ph.D (Analysis, ITB-Bandung, Indonesia)
- Prof. Dr. Edy Hermawan (Atmospheric Modeling, LAPAN Bandung)
- Prof. H. Yaya S. Kusumah, M.Sc., Ph.D (Mathematics Education, UPI-Bandung, Indonesia)
- Prof. Dr. Slamin (Combinatorics, Universitas Jember, Indonesia)
- Dr. Aleams Barra (Algebra, ITB-Bandung, Indonesia)

We thank all who sent the papers or proceedings of IICMA 2013. We also would like to give our gratitude for all reviewers who worked hard for making this proceedings done.

IndoMS conveys high appreciation for the Directorate General of Higher Education (DGHE) for the most valuable support in organizing the conference. We also would like to give our gratitude to Universitas Gadjah Mada, especially to Department of Mathematics, Faculty of Mathematics and Natural Sciences for providing the places and staffs for this conference.

It remains to thank all members of Organizing Committee spread across 3 cities, Depok, Bandung and Yogyakarta who have worked very hard to make this conference happens.

Yogyakarta, January 5th, 2014
On behalf of the Committee
Dr. Kiki Ariyanti Sugeng - Chair.

ACKNOWLEDGEMENT

The organizing Committee of the IICMA 2013 and Indonesian Mathematical Society (IndoMS) wish to express their gratitude and appreciation to all Sponsors and Donors for their help and support for the Program, either in form of financial support, facilities, or in other form. The Committee addresses great thank especially to:

a. Directorate General of Higher Education (DGHE)
b. The Rector of the Gadjah Mada University
c. The Dean of Faculty ofMathematics and Natural Sciences, Gadjah Mada University.
d. The Head of the Department of Mathematics, Gadjah Mada University.
e. All sponsors of the conference

The Committee extends its gratitude to all invited speakers, parallel session speakers, and guests for having kindly and cordially accepted the invitation and to all participants for their enthusiastic response.

Finally the Committee also would like to acknowledge and appreciate for the support and help of all IndoMS members in the preparation for and the running of the program

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COMPARISON OF SENSITIVITY ANALYSIS ON LINEAR OPTIMIZATION USING OPTIMAL PARTITION AND OPTIMAL BASIS (IN THE SIMPLEX METHOD) AT SOME CASES

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Abstract. Sensitivity analysis describes the effects of coefficient changes of a linear optimization problem to the optimal solution. Usually we use the optimal basis approach as in the simplex method. This paper discussed the sensitivity analysis with another approaches: analysis using an optimal partition based on the interior point method to determine the range and shadow price. We then compare the results obtained with those produced by the simplex method with the help of software LINDO 6.1. The results of sensitivity analysis, obtained through the optimal partition approach is more accurate than using the optimal basis approach (the simplex method), especially for cases where the primal or the dual optimal solution is not unique. But when the primal and the dual have a unique optimal solution, the simplex method and the optimal partition approach produce same information.

Key words and Phrases : sensitivity analysis, shadow price, range, optimal partition, optimal basis.

1. Introduction

Linear Optimization (LO) is concerned with the minimization or maximization of a linear function, subject to constraints described by linear equations and/or linear inequalities.

Sensitivity analysis describes the effect of changing the parameters of the linear optimization model, i.e. studying the effect of changing the coefficients of objective function and right-hand side value constraints to the optimal solution. Sensitivity analysis that is used in the classical approach (the simplex method) based on the optimal basis. This paper will present briefly sensitivity analysis by using another approach, the analysis using the unique partition (optimal partition) based on the interior point method. This method is presented by Roos, Terlaky and Vial [1]. By using the optimal partition approach, we determine shadow price and range. For the same problem we also performed a sensitivity analysis using the simplex method with the help of software LINDO 6.1. Then we compare the obtained results.

The structure of this paper is as follows. In section 2, we review shortly the primal-dual problem, optimal partition and optimal sets, range and shadow price, and sensitivity analysis with classical approach. In section 3, we present three cases of LO problems to be analyzed and compared by using optimal partition and by using LINDO 6.1. At the end we give concluding remarks.

2. Sensitivity Analysis

2.1. Primal - Dual

Every linear optimization problem can be modeled mathematically into a form called the primal form (P) and the dual form (D).

The standard form of a primal and a dual form are as follows:
(P) $\min \{c^T x : Ax = b, x \geq 0\}$,
(D) $\max \{b^T y : A^T y + s = c, s \geq 0\}$,
where $c, x, s \in \mathbb{R}^n$, $b, y \in \mathbb{R}^m$ and $A \in \mathbb{R}^{m \times n}$ is matrix with rank m.

Suppose the optimal value of (P) and (D) symbolized by $v(b)$ and $v(c)$:
$v(b) = \min \{c^T x : Ax = b, x \geq 0\}$,
$v(c) = \max \{b^T y : A^T y + s = c, s \geq 0\}$.
The feasible regions of (P) and (D) are denoted by P and D, respectively:
$P := \{x \in \mathbb{R}^n : Ax = b, x \geq 0\}$,
$D := \{(y, s) \in \mathbb{R}^m : A^T y + s = c, s \geq 0\}$.

If (P) and (D) are feasible then both problems have optimal solutions, and we denote it by P^* and D^*,
$P^* := \{x \in P : c^T x = v(b)\}$
$D^* := \{(y, s) \in D : b^T y = v(c)\}$.

2.2. Optimal Partition and optimal sets

The followings are the theorems that used as base of forming an optimal partition.

Theorem 1. *(Duality Theorem, cf.* [1] *Theorem II.2) If (P) and (D) are feasible then both problems have optimal solutions. Then, if $x \in P$ and $(y, s) \in D$, these are optimal solutions if and only if $x^T s = 0$. Otherwise neither of the two problems has optimal solutions, either both (P) and (D) are infeasible or one of the two problems is infeasible and the other one is unbounded.*

Theorem 2. *(Goldman-Tucker, cf.* [1] *Theorem II.3) If (P) and (D) are feasible then there exists a strictly complementary pair of optimal solutions, that is an optimal solution pair (x^*, s^*) satisfying $x^* + s^* > 0$.*

The optimal partition of (P) and (D) are the partition that splits the index of x (and s) into B and N, as follows:

$$B := \{i : x_i > 0 \text{ for some } x \in P^*\},$$
$$N := \{i : s_i > 0 \text{ for some } (y, s) \in D^*\}.$$

We may check that the duality theorem implies $B \cap N = \emptyset$, and Goldman-

Tucker theorem implies $B \cup N = \{1, 2, ..., n\}$.

We use x_B and x_N as notations refer to the restriction of the vector $x \in \mathbb{R}^n$ to the index set B and N respectively. Similarly, A_B and A_N represent the restriction of A to the columns with indices of set B and N respectively. We then have the following lemma.

Lemma 1.(*cf.* [1]) *P^* and D^* can be expressed by the terms of the optimal partition into*

$$P^* = \{x : Ax = b,\ x_B \geq 0,\ x_N = 0\},$$
$$D^* = \{(y, s) : A^T y + s = c,\ s_B = 0,\ s_N \geq 0\}.$$

2.3. Range and Shadow Price

Sensitivity analysis determines the shadow price and range of all the coefficients b (the value of the right side of primal constraints) and c (the value of the right side dual constraints). In one case, the value of coefficient b or c may be a break point. If the coefficient is a break point, then we have two shadow prices: the left shadow price and right shadow price. If the coefficient is not a break point, then there is a shadow price at an open linearity interval and range of the coefficient is in the linearity interval. Figure 1 shows an example of change in the optimal value for the change in the value of cj (c_j=1 and c_j=2 are break points).

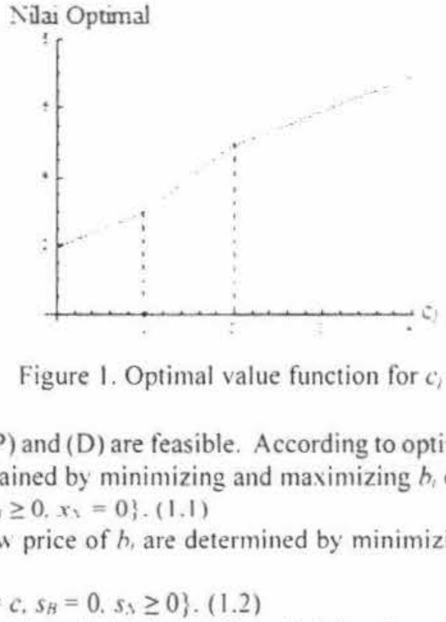

Figure 1. Optimal value function for c_j

Suppose that (P) and (D) are feasible. According to optimal partition approach [1], range of b_i is obtained by minimizing and maximizing b_i over the set

$\{b_i : Ax = b,\ x_B \geq 0,\ x_N = 0\}$. (1.1)

Left and right shadow price of b_i are determined by minimizing and maximizing y_i over the set

$\{y_i : A^T y + s = c,\ s_B = 0,\ s_N \geq 0\}$. (1.2)

Range of c_j is obtained by minimizing and maximizing the value of c_j over the set

$\{c_j : A^T y + s = c,\ s_B = 0,\ s_N \geq 0\}$. (1.3)

Left and right shadow price of c_j are determined by minimizing and maximizing x_j over the set

$\{x_j : Ax = b,\ x_B \geq 0,\ x_N = 0\}$. (1.4)

2.4. Sensitivity Analysis with the Classical Approach

Sensitivity analysis with the classical approach based on the simplex method to solve linear optimization problems. The optimal solution of this classical approach is determined by an optimal basis.

Assume that A is a matrix of size mxn and rank $(A) = m$. Indices of a basis variable is denoted by B'. Then sub-matrix $A_{B'}$ is a non-singular matrix of size mxm with $A_{B'}x_{B'}=b$, $x_{N'}=0$ where N' is the set of non-basis variable index of A. A primal basic solution x can be determined by

$$x = \begin{pmatrix} x_{B'} \\ x_{N'} \end{pmatrix} = \begin{pmatrix} A_{B'}^{-1}b \\ 0 \end{pmatrix}. \quad (1.5)$$

and a dual basic solution can be determined by

$$y = A_{B'}^{-T}c_{B'}, \; s = \begin{pmatrix} s_{B'} \\ s_{N'} \end{pmatrix} = \begin{pmatrix} 0 \\ c_N - A_{N'}^T y \end{pmatrix}. \quad (1.6)$$

Sensitivity analysis with the classical approach uses also formulas (1.5) - (1.8) to determine the range and shadow price, but with the optimal basis partition (B', N') instead of (B, N). In fact, P and D may have more than one optimal basis, and therefore this classical approach may also provides different shadow price and range [2].

3. Cases

We consider three cases as follows:

1. Optimal solution of the primal problem is unique and optimal solution of the dual problem is not unique.
2. Optimal solution of the primal problem is not unique and optimal solution of the dual problem is unique.
3. Optimal solution of the primal and the dual problems are unique.

3.1. Case I

Suppose the primal problem (P) is defined as follows:

Min$4x_1 - 5x_2 + 11x_3$

s.t $-x_2 + 3x_3 = 0$

$x_1 - x_2 - x_3 = 1$

$x_1, x_2, x_3 \geq 0$

The dual problem (D) is

Maxy_2

s.t$y_2 \leq 4$

$-y_1 - y_2 \leq -5$

$3y_1 - y_2 \leq 11$

The feasible region of the dual problem is depicted in Figure 2. From Figure 2. it can be seen that the set of optimal solutions of (D) is $D^* = \{(y_1, y_2): 1 \leq y_1 \leq 5, y_2 = 4\}$ and the optimal value is 4 . Slack variable of each of the dual constraints are as follows:

$$y_2 + s_1 = 4 \Leftrightarrow s_1 = 4 - y_2$$
$$-y_1 - y_2 + s_2 = -5 \Leftrightarrow s_2 = -5 + y_1 + y_2$$
$$3y_1 - y_2 + s_3 = 11 \quad \Leftrightarrow s_3 = 11 - 3y_1 + y_2$$

It can be concluded that all the slack can be positive at an optimal solution unless the slack value of the constraint $y_2 \leq 4$, i.e. $s_1 = 0$. This means that the optimal partition of set N is $N=\{2, 3\}$. Hence $B=\{1\}$.
By using Lemma 1, we get:
$P^* = \{x \in P: x_2 = x_3 = 0\}$ and (P) has a unique solution $x = (1, 0, 0)$.

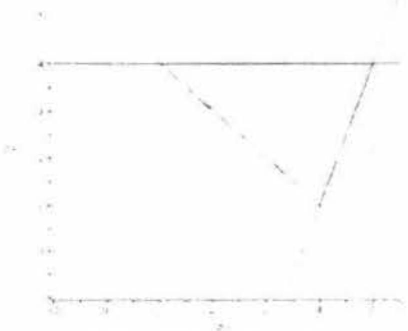

Figure 2. Feasible region of case I.

Next we show examples of finding range and shadow price of $b_1 = 0$ and $c_1 = 4$. The other range and shadow price can be found in the same way.

Range and Shadow Price for $b_1 = 0$
By using (1.1), range b_1 can be determined by minimizing and maximizing b_1 over the set $\{b_1: Ax = b, x_B \geq 0, x_N = 0\}$.
We have $Ax = b$ as follows

$$\begin{bmatrix} 0 & -1 & 3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} b1 \\ 1 \end{bmatrix}.$$

From the above system we get

$$0 = b_1$$
$$x_1 = 1.$$

Hence the range of b_1 is the interval [0, 0]. Therefore $b_1 = 0$ is a break point.
By using (1.2), the shadow price of b_1 can be determined by minimizing and maximizing y_1 over the set $\{y_1 : A^T y + s = c, s_B = 0, s_N \geq 0\}$.
Using that $y \in D^*$, the minimum value of y_1 is 1 and the maximum value is 5, so the shadow price for b_1 is [1, 5].

Range and Shadow Price for $c_1 = 4$
Range of c_1 determine by minimizing and maximizing c_1 over the set $\{c_1: A^T y + s = c, s_B = 0, s_N \geq 0\}$, as in (1.3).
Matrix multiplication of $A^T y + s = c$:

$$\begin{bmatrix} 0 & 1 \\ -1 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} y1 \\ y2 \end{bmatrix} + \begin{bmatrix} s1 \\ s2 \\ s3 \end{bmatrix} = \begin{bmatrix} c1 \\ -5 \\ 11 \end{bmatrix}$$

Based on Figure 1, if we eliminate the first constraint, y_2 will be in the interval $[1, \infty)$. By substituting $s_1 = 0$ and y_2 to the first constraint, we get $y_2 = c_1$. This means that $1 \leq c_1 \leq \infty$, hence the range for c_1 is the interval $[1, \infty)$.
By using (1.4) shadow price of c_1 is determine by minimizing and maximizing x_1 over the set $\{x_1: Ax = b, x_B \geq 0, x_N = 0\}$. Because of $x_1 = 1$, then the shadow price of c_1 is 1.

In Table 1, we present range and shadow price of case I which are obtained by using optimal partition approach. We also present range and shadow price obtained from calculation by using LINDO (Table 2). We may see sensitivity analysis of the simplex method (LINDO) did not detect that $b_1 = 0$ is a break point.

Table 1. Range and shadow price obtained from optimal partition approach (Case I)

Coefficient	Range	Shadow price
$b_1 = 0$	0	[1, 5]
$b_2 = 1$	$[0, \infty)$	4
$c_1 = 4$	$[1, \infty)$	1
$c_2 = -5$	$[-9, \infty)$	0
$c_3 = 11$	$[-1, \infty)$	0

Table 2. Range and shadow price obtained from LINDO (Case I)

Coefficient	*Range*	*Shadow price*
$b_1 = 0$	$(-\infty, 0]$	1
$b_2 = 1$	$[0, \infty)$	4
$c_1 = 4$	$[1, \infty)$	1
$c_2 = -5$	$[-9, \infty)$	0
$c_3 = 11$	$[-1, \infty)$	0

3.2. Case II

Suppose the primal problem (P) is defined as follows:
Min $4x_1 + 31x_2 - 5x_3 + 11x_4$
s.t $3x_2 - x_3 + 3x_4 = 0$
$x_1 + 7x_2 - x_3 - x_4 = 1$
$x_1, x_2, x_3, x_4 \geq 0.$

The dual problem (D) is
Max y_2
s.t $y_2 \leq 4$
$3y_1 + 7y_2 \leq 31$
$-y_1 - y_2 \leq -5$
$3y_1 - y_2 \leq 11$

The feasible region of the dual problem is shown in Figure 3. From Figure 3, we obtain that the optimal solution of (D) is $D^* = \{(y_1, y_2): y_1 = 1, y_2 = 4\}$ and the optimal value is 4. Slack variable of each of the dual constraints are as follows:

$$y_2 + s_1 = 4 \Leftrightarrow s_1 = 4 - y_2$$
$$3y_1 + 7y_2 + s_2 = 31 \Leftrightarrow s_2 = 31 - 3y_1 - 7y_2$$
$$-y_1 - y_2 + s_3 = -5 \Leftrightarrow s_3 = -5 + y_1 + y_2$$
$$3y_1 - y_2 + s_4 = 11 \Leftrightarrow s_4 = 11 - 3y_1 + y_2$$

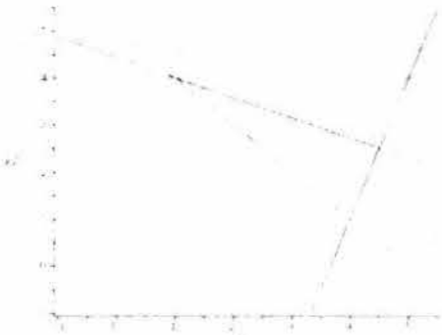

Figure 3. Feasible region of case II.

By substituting $y_1 = 1$, $y_2 = 4$, we obtain the values of each slack. Slack in these constraints: $y_2 \le 4$, $3y_1 + 7y_2 \le 31$ and $-y_1 - y_2 \le -5$ are 0, i.e. $s_1 = s_2 = s_3 = 0$. Hence in the primal problem only x_1, x_2 and x_3 can be positive. Therefore the optimal partition (B, N) is obtained, i.e. N = {4} and B = {1, 2, 3}.

By using Lemma 1, we get:
$P^* = \{x \in P: x_4 = 0\}$ and (P) has not unique solution : $\{ (x_1\ x_2\ x_3) : (a\ ,\ \frac{1}{4} - \frac{1}{4}a,\ 3\ (\frac{1}{4} - \frac{1}{4}a)) \}$, $0 \le a \le 1$.
By using the same calculation as in case I, we get ranges and shadow prices of case II (Table 3). Table 4 shows ranges and shadow prices of case II obtained by using LINDO.

Table 3. Range and shadow price obtained from optimal partition approach (Case II).

Coefficient	*Range*	*Shadow price*
$b_1 = 0$	$(-\infty, 3/7]$	1
$b_2 = 1$	$[0, \infty)$	4
$c_1 = 4$	4	[1, 0]
$c_2 = 31$	31	[¼, 0]
$c_3 = -5$	-5	[¾, 0]
$c_4 = 11$	$[-1, \infty)$	0

Table 4. Range and shadow price obtained from LINDO (Case II).

Coefficient	*Range*	*Shadow price*
$b_1 = 0$	$(-\infty, 0]$	1
$b_2 = 1$	$[0, \infty)$	4
$c_1 = 4$	$[1, 4]$	1
$c_2 = 31$	$[31, \infty)$	0
$c_3 = -5$	$[-5, \infty)$	0
$c_4 = 11$	$[-1, \infty)$	0

From Table 3 and Table 4, there are differences in range and shadow price obtained by using optimal partition and the simplex method. At the coefficient $b_1 = 0$, for the same shadow price, the optimal partition detect a greater range. Next, at the coefficients $c_1 = 4$, $c_2 = 31$, and $c_3 = -5$, analysis using the simplex method does not detect any break points.

3.3. Case III

Suppose the primal problem (P) is defined as follows:

$\text{Min} 31x_1 - 5x_2 + 11x_3$

s.t $3x_1 - x_2 + 3x_3 = 0$

$7x_1 - x_2 - x_3 = 1$

$x_1, x_2, x_3 \geq 0$

Dual problem (D) is

Max y_2

s.t$3y_1 + 7y_2 \leq 31$

$-y_1 - y_2 \leq -5$

$3y_1 - y_2 \leq 11$

The feasible region of the dual problem is shown in Figure 4. From Figure 4, it can be determined that the optimal solution of (D) is $D^* = \{(y_1, y_2): y_1 = 1, y_2 = 4\}$ and the optimal value is 4 . Slack variable of each of the dual constraints are as follows:

$$3y_2 + 7y_2 + s_1 = 31 \Leftrightarrow s_1 = 31 - 3y_1 - 7y_2$$
$$-y_1 - y_2 + s_2 = -5 \Leftrightarrow s_2 = -5 + y_1 + y_2$$
$$3y_1 - y_2 + s_3 = 11 \quad \Leftrightarrow s_3 = 11 - 3y_1 + y_2$$

We can check that at $y_1 = 1$ and $y_2 = 4$, all the slack can be positive except slack in the constraint $3y_1 + 7y_2 \leq 31$ and $-y_1 - y_2 \leq -5$, at the constraints mentioned we have $s_1 = s_2 = 0$. Hence the optimal partition (B, N) is $N = \{3\}$ and $B = \{1, 2\}$.

By using Lemma 1, we obtain:

$P^* = \{x \in P: x_3 = 0\}$ and (P) has a unique solution $x = (\frac{1}{4}, \frac{3}{4}, 0)$.

By using the same calculation as before, we get ranges and shadow prices of case III (Table 5). Table 6 shows ranges and shadow prices of case III obtained by using LINDO.

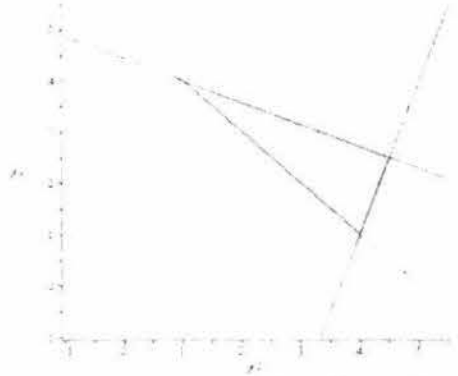

Figure 4. Feasible region of case II

Table 5. Range and shadow price obtained from optimal partition approach (Case III).

Coefficient	*Range*	*Shadow price*
$b_1 = 0$	$(-\infty, 3/7]$	1
$b_2 = 1$	$[0, \infty)$	4
$c_1 = 31$	$[19, \infty)$	$1/4$
$c_2 = -5$	$[-7, \infty)$	$3/4$
$c_3 = 11$	$[-1, \infty)$	0

Table 6. Range and shadow price obtained from LINDO (Case III).

Coefficient	*Range*	*Shadow price*
$b_1 = 0$	$(-\infty, 3/7]$	1
$b_2 = 1$	$[0, \infty)$	4
$c_1 = 31$	$[19, \infty)$	$1/4$
$c_2 = -5$	$[-7, \infty)$	$3/4$
$c_3 = 11$	$[-1, \infty)$	0

We may see that the range and shadow price using optimal partitioning and the simplex method are same.

4. Concluding Remarks

The results of sensitivity analysis by using the simplex method (using the optimal basis approach) for cases where one of the primal or dual optimal solution is not unique, is not as perfect as the results obtained by using optimal partition approach. When the primal and the dual have a unique optimal solution, simplex method and optimal partition approach give the same information.

References

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