ISBN 978-979-95093-9-0

2013 ISS INTERNATIONAL SEMINAR ON SCIENCES FMIPA IPB

PROCEEDINGS

INTERNATIONAL SEMINAR ... 2013

FACULTY OF MATHEMATICS AND NATURAL SCIENCES,
BOGOR AGRICULTURAL UNIVERSITY
IPB International Convention Center
15 - 17th November 2013

Published By

ISBN : 978-979-95093-9-0

PROCEEDINGS

ISS 2013

International Seminar on Sciences 2013
"Perspectives on Innovative Sciences"

Bogor 15-17 November 2013

IPB International Convention Center

Published by

Faculty of Mathematics and Natural Sciences
Bogor Agricultural University

Proceedings of International Seminar on Sciences 2013 "Perspectives on Innovative Sciences"
Bogor 15-17 November 2013.
Published by: FMIPA-IPB, Jalan Meranti Kampus IPB Dramaga, Bogor 16680
Telp/Fax: 0251-8625481/8625708
http://fmipa.ipb.ac.id
ix + 395 pages
ISBN: 978-979-95093-9-0

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FOREWORD

The International Seminar on Sciences 2013, which had the main theme "Perspectives on Innovative Sciences", was organized on November 15th -17th, 2013 by the Faculty of Mathematics and Natural Sciences, Bogor Agricultural University. This event aimed at sharing knowledge and expertise, as well as building network and collaborations among scientists from various institutions at national and international level.

Scientific presentations in this seminar consisted of a keynote speech, some invited speeches, and about 120 contributions of oral and poster presentations. Among the contributions, 66 full papers have been submitted and reviewed to be published in this proceeding. These papers were clustered in four groups according to our themes:

A. Sustainability and Science Based Agriculture

B. Science of Complexity

C. Mathematics, Statistics and Computer Science

D. Biosciences and Bioresources

In this occasion, we would like to express our thanks and gratitude to our distinguished keynote and invited speakers: Minister of Science and Technology, Prof. Manabu D. Yamanaka (Kobe University, Japan), Prof. Kanaya (Nara Institute of Science and Technology, NAIST, Japan), Prof. Ken Tanaka (Toyama University, Japan), Emmanuel Paradis, PhD. (Institut de Recherche pour le Développement, IRD, France), Prof. Dr. Ir. Rizaldi Boer, MS (Bogor Agricultural University), and Prof. Dr. Ir. Antonius Suwanto, M.Sc. (Bogor Agricultural University).

We would like also to extend our thanks and appreciation to all participants and referees for the wonderful cooperation, the great coordination, and the fascinating efforts. Appreciation and special thanks are addressed to our colleagues and staffs who help in editing process. Finally, we acknowledge and express our thanks to all friends, colleagues, and staffs of the Faculty of Mathematics and Natural Sciences IPB for their help and support.

Bogor, March 2014

The Organizing Committee

International Seminar on Sciences 2013

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Gap between the Lower and Upper Bounds for the Iteration Complexity of Interior-Point Methods

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Abstract

Recently, the use of interior-point methods to solve linear optimization problems, have been becoming great attention to the researchers. The most important thing is that the interior point methods have the best complexity compared to other methods and also efficient in practice. Gonzaga, Monteiro and Adler presented small-update path-following methods, a variant of interior-point methods, which is the best known upper bound for the iteration complexity of an interior-point method. Roos, Terlaky and Vial presented an interior-point method using primal-dual full-Newton step algorithm and state the upper bound for the iteration complexity of an interior-point method in difference expression. Deza, Nematollahi, Peyghami and Terlaky showed several worst cases of the interior-point method by using Klee-Minty problem. Using their worst cases, we present a lemma, where from this lemma we may obtain the lower bound for the iteration complexity of an IPM.

Keywords: *interior-point method, upper bound, lower bound*

I. Introduction

Optimization is the branch of applied mathematics which studies problems where one seeks to minimize (or maximize) a real function of real variables, subject to constraints on the variables. The solution set of the constraints defines the feasible region (or the domain) of an optimization problem.

II. Brief History of Linear Optimization

Linear Optimization (LO) is concerned with the minimization or maximization of a linear function, subject to constraints described by linear equations and/or linear inequalities.

A. Simplex Methods

LO emerged as a mathematical model after World War II, when Dantzig in 1947 proposed his simplex method for solving "linear programming" (then known as optimization) problems [1].

The feasible region of an LO problem is a polyhedron, the solution set to a system of linear constraints. Simplex methods move along vertices of the polyhedron in order to find an optimal vertex. These methods are designed in such a way that during this process the value of the objective function changes monotonically to its optimal value.

After its discovery, the Dantzig simplex method has inspired much research in mathematics. Simplex methods were placed as the top 10 algorithms in the twentieth century by the journal Computing in Science and Engineering [2]. There are many variants of a simplex method, distinguished by rules for selecting the next vertex (so-called pivot rules). The success of simplex methods have raised some questions such as: whether there exists a pivot rule that requires a polynomial number of iterations, and whether there are linear optimization problems that require an exponential number of iterations.

The last question was answered by Klee and Minty [3] in 1972. They gave an example of an LO problem with $2n$ inequalities for which the simplex method may need as much as 2^n-1 iterations. Their example uses Dantzig's classic most-negative-reduce-cost pivot rule.

The n-dimensional Klee-Minty (KM) problem is given by :

$\min y_n$

subject to $\rho y_{k-1} \le y_k \le 1-\rho y_{k-1},\ k=1,\dots,n,$

(26)

where ρ is small positive number by which the unit cube $[0,1]^n$ is squashed, and $y_0=0$. The domain is a perturbation of the unit cube in R^n. If $\rho=0$ then the domain is the unit cube and for $\rho\in(0,1/2)$ it is a perturbation of the unit cube which is contained in the unit cube itself, as can easily be verified. Since the perturbation is small, the domain has the same number of vertices as the unit cube, i.e. 2^n. Klee and Minty showed that in their example the simplex method with the Dantzig rule walks along all these vertices. Thus it became clear that the computational time needed by the Dantzig simplex method may grow exponentially fast in terms of the number of inequalities. Since then exponential examples have been found for almost every pivot rule.

B. The ellipsoid Method

The shortcomings of simplex methods (at least theoretically) stimulated researchers to look for other methods with a running time that grows polynomially fast if the number of inequalities grows. The first polynomial-time algorithm for LO problems is the ellipsoid method. The basic ideas of this method evolved from research done in the 1960s and 1970s in the Soviet Union (as it then was called). The idea of the ellipsoid method is to enclose the region of interest by an ellipsoid and to decrease the volume of the ellipsoid in each iteration [4]. This method was first published in a paper of Iudin and Nemirovskii [5]. Independently in 1977, Shor [6] also presented the ellipsoid method. Khachiyan modified this method and in 1979 [7] he introduced this method as the first polynomial-time algorithm for LO problems.

He proved that the ellipsoid method solves an LO problem in $O(n^2L)$ iterations with the total number of arithmetic operations $O(n^5L)$, where n is the number of inequalities and L is the total bit-length of the input-data. Then in his next paper [8] he gave a better bound, $O(n^4L)$, for the total number of arithmetic operations.

Following Khachiyan's work, the ellipsoid method was studied intensively for its theoretical and practical aspects, with the hope that LO problems could be solved faster than by simplex methods. The results were not as expected. In practice, the rate of convergence of the ellipsoid method is rather slow, when compared to simplex methods. The worst-case iteration bound for simplex methods, in any of its several implementations, is an extremely poor indicator of the method's actual performance. On the other hand, the worst-case bound for the ellipsoid method appears to be a good indicator for the practical behavior of the ellipsoid method [9], which makes the method become too slow for practical purposes.

C. Interior-point Methods

A really effective breakthrough occurred in 1984, when Karmarkar [10] proposed a different polynomial-time method (known as Karmarkar's projective method) for LO problems. Contrary to simplex methods, whose iterates are always on the boundary of the domain, Karmarkar's method passes through the interior of the domain to find an optimal solution.

In the worst-case, for a problem with n inequalities and L bits of input data, his method requires $O(nL)$ iterations. In each iteration, Karmarkar's algorithm requires $O(n^{2.5})$ arithmetic operations and each arithmetic operation needs a precision of $O(L)$ bits. In total, in the worst-case, Karmarkar's algorithm requires $O(n^{3.5}L)$ arithmetic operations on numbers with $O(L)$ bits. The theoretical running time of this algorithm is better than that of the ellipsoid algorithm by a factor of $O(\sqrt{n})$. More exciting, Karmarkar claimed that the algorithm is not only efficient in theory, but also in practice.

Karmarkar's paper initiated a revolution in the field of optimization. It gave rise to so-called interior-point methods (IPMs), first for LO but later also for the more general class of convex problems.

Renegar, in 1988 [11], improved the number of iterations to $O(\sqrt{n}L)$ iterations. Other variants of IPMs, called potential reduction methods, require also only $O(\sqrt{n}L)$ iterations. This was shown by Ye [12], Freund [13], Todd and Ye [14] and Kojima, Mizuno and Yoshise [15]. The main idea of these methods is the usage of a potential function for measuring the progress of the method. Karmarkar's projective method also uses a potential function.

A wide class of IPMs uses the so-called *central path*, which was introduced by Sonnevend [16] and Meggido [17], as a guide line to the set of optimal solutions; these methods are therefore called path-following methods. Small-update path-following methods, a variant of path-following methods, were presented Monteiro and Adler [18] and Roos and Vial [19]. Their methods require $O(\sqrt{n}L)$ iterations, which is the best known upper bound for the iteration complexity of an IPM. Roos, Terlaky and Vial in their book [20] obtained the same upper bound by using an algorithm which is a so-called primal-dual full-Newton step algorithm. Expressing the absolute accuracy of the objective function by ε their upper bound for the number of iterations is

$$\left\lceil \sqrt{2n}\ln\frac{n\mu^0}{\varepsilon} \right\rceil, \quad (27)$$

where $\mu^0 > 0$ denotes the initial value of the so-called barrier parameter.

III. Gap between the Lower and Upper Bounds

Figure 1 shows the central path of the 2-dimensional Klee-Minty (KM) problem for $\rho = 1/3$.

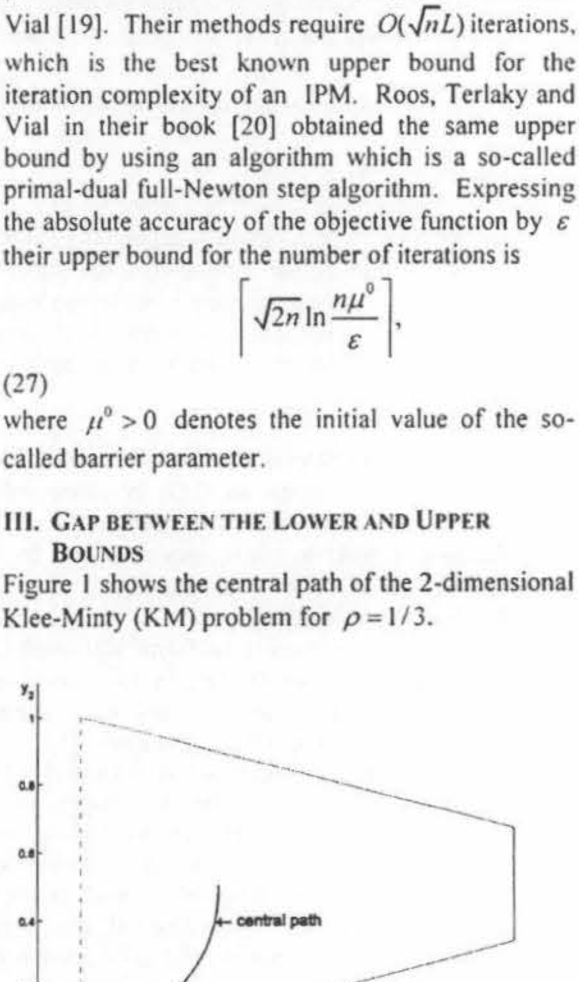

Figure 1. Central path of the 2-dimensional KM problem, $\rho = 1/3$.

The central path is an analytic curve that moves through the interior of the domain to the optimal set.

Ideally it is a nice smooth curve that goes more or less straightforward to the optimal set. In that case path-following methods are extremely efficient. In practice the curve may possess some sharp turns. We assume that each such turn will require at least one iteration of a path-following IPM. As a consequence we may consider the number of sharp turns in the central path as a lower bound for the number of iteration of any path-following IPM.

Recently, Deza, Nematollahi, Peyghami and Terlaky [21] showed that when adding abundantly many suitable chosen redundant constraints to the KM cube, the squashed cube which is formed by the inequalities of KM problem, then one may force the central path to visit small neighborhoods of all the vertices of the KM cube.

Deza et al. concluded that an IPM needs at least 2^n-1 iterations to solve their problem. Hence the number of iterations may be exponential in the dimension n of the cube. This does not contradict the polynomial-time iteration bound (2) however. Note that the number of inequalities of the n-dimensional KM problem is $2n$. If N denotes the number of inequalities in the problem that causes the central path to visit small neighborhoods of all the vertices of the KM cube, then the upper bound should be $O\left(\sqrt{N}\ln(N\mu^0/\varepsilon)\right)$. Assuming that $\mu^0/\varepsilon=O(1)$, one may consider this bound as $O\left(\sqrt{N}\ln N\right)$. The bound implies that the number N of inequalities must be exponential in n as well, because we should have $2^n-1=O\left(\sqrt{N}\ln N\right)$.

In [21], Deza et al. also argued that such a redundant KM problem, whose central path visits small neighborhoods of all the vertices of the KM cube, gives rise to a lower bound for the maximal number of iterations in terms of N. Several papers appeared since then, each new paper using less redundant constraints and, as a consequence, yielding a higher lower bound for the iteration complexity of an IPM. The results of these papers are summarized in Table 1.

Table 1. Results from the literature.

Type of redundant constraints	Number of redundant inequalities	Lower bound for iteration complexity	Reference
$\rho y_{k-1}-y_k\le d$	$O(n^2 2^{6n})$	$\Omega\left(\sqrt[6]{\frac{N}{\ln^2 N}}\right)$	[21]
$\rho y_{k-1}-y_k\le d$	$O(n2^{3n})$	$\Omega\left(\sqrt[3]{\frac{N}{\ln N}}\right)$	[22]
$\rho y_{k-1}-y_k\le d$	$O(n^3 2^{2n})$	$\Omega\left(\sqrt{\frac{N}{\ln^3 N}}\right)$	[23]
$-y_k\le d_k$	$O(n2^{2n})$	$\Omega\left(\sqrt{\frac{N}{\ln N}}\right)$	[24]

Column 4 gives the related references. Column 1 shows the type of constraints used in the corresponding paper, column 2 the order of the number N of inequalities used, and column 3 the resulting lower bound for the number of iterations of IPMs. In each case one has $N\ge 2^n$.

The next lemma explains how the lower bounds in column 3 can be deduced from the figures in column 2 of Table 1.

Lemma 1. *If the number N of inequalities describing a redundant KM problem is $O(n^p 2^{qn})$ and the central path enters a small neighborhood of each vertex, then any IPM requires at least*

$$\Omega\left(\sqrt[q]{\frac{r^p N}{\ln^p N}}\right)$$

iterations, where r is such that $N\ge 2^{rn}$ and $p,q,r>0$.

Proof:
It is well known that if an iterate x is on (or close to) the central path, then the search direction at x in any interior-point method is (about) tangent to the central path. The KM path consists of 2^n-1 line segments. Since for each sharp turn in the central path an IPM requires at least one iteration, when solving the redundant KM problem at least 2^n-1 iterations are needed. The number N of inequalities being $O(n^p 2^{qn})$, we have $N\le cn^p 2^{qn}$ for some $c>0$. This implies $\frac{N}{cn^p}\le 2^{qn}$ for some $c>0$. Thus we obtain

$$\#\text{ iterations }+1\ge 2^n\ge\left(\frac{N}{cn^p}\right)^{\frac{1}{q}}.$$

From $N \geq 2^m$ we derive, by taking the 2-logarithm at both sides, that $\log_2 N \geq rn$, whence $n \leq \log_2 N / r$. Substituting this we get

$$\# \text{ iterations } + 1 \geq \left(\frac{Nr^p}{c(\log_2 N)^p} \right)^{\frac{1}{q}}.$$

This implies the statement in the lemma. □

As stated before, the best known upper bound for the iteration complexity of an IPM is the bound in (27). For fixed values of μ^0 and ε we may write

$$\sqrt{N} \ln \frac{N\mu^0}{\varepsilon} = \sqrt{N} \ln N + \sqrt{N} \ln \frac{\mu^0}{\varepsilon}$$
$$= O(\sqrt{N} \ln N).$$

Comparing this with the highest lower bound in Table 1, which is $\Omega\left(\sqrt{N / \ln N}\right)$, we conclude that there is still a gap between the lower and upper bounds for the iteration complexity of IPMs: the bounds differ by a factor $\ln^{\frac{3}{2}} N$.

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