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BUSES DISPATCHING PROBLEM IN URBAN TRANSPORT SYSTEM

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Abstract

The complexity of transport system in urban area will significantly increase in accordance with demographic development and urban transport system. The transport system implies a higher transport cost incurred by fuel wasting due to traffic jam, productivity deterioration and environmental degradation. Busway as one of transport modes in Bus Rapid Transit (BRT) system is commonly adopted by local government to circumvent such kind of drawbacks, since it may reassign the use of private cars into public transport. However, in some cases, the implementation of busway system is not optimal due to poor planning. This paper develops a deterministic mathematical model for finding the optimal number of buses to be assigned in busway system.
framework to model the operation of Transjakarta, busway transport system in the metro city of Jakarta. An integer programming is established to determine the optimum number of dispatched buses from the initial shelter under minimum operational cost. Optimum dispatching leads to a minimum waiting time and assures passengers’ comfort.

1. Introduction

Urban areas are locations having a high level of accumulation and concentration of economic activities and are complex spatial structures. Bus transport in big cities and other developed areas has high complexity due to the modes involved, the multitude of origins and destinations, the amount and variety of traffic including congestion and the agility of public transportation systems, and the continuous growth of urban population. These major challenges are experienced not only by households and businesses, but also by the urban community at large. Therefore, transport may become a binding constraint on both economic and social aspects, along with enlarged negative impacts on health and on the environment. According to [7], one of the most notable urban transport problems is the public transport inadequacy, where it mainly relates to the overuse of public transit systems. During busy hours, crowdedness creates discomfort for passengers as the system copes with a temporary surge in demand, but in the other side, low ridership makes many services financially unsustainable.

The dispatching problem is a dynamic decision problem commonly encountered in transport system as well as in manufacturing sites, batch jobs in computing and web server farms. In this paper, we present a buses dispatching model with primary objective is to minimize the number of dispatched buses from initial shelter each time-slot. Instead of treating the service demand as a given continuous function, we split the time horizon into discrete time-slots and determine the dispatching rate for each of these intervals.
2. Bus Dispatching Model

1. Problem statement and assumptions

In the present work, we consider a busway route or corridor consisting of number of shelters or bus-stops served by company which has a number of types, each bus has certain capacity and operational cost. We aim to analyze the relationship between the number of embarked passengers and the number of dispatched buses. Particularly we attempt to determine the minimum number of dispatched buses from initial shelter each time-slot. Information: the maximum number of passengers under minimum operational cost can so be provided. We made the following assumptions in order to simplify the analysis: (1) the track of busway is secured from other vehicles such that there is no obstacle during the operation of busway, (2) time for fuel supply and time-stop due to traffic light is ignored, (3) buses are homogeneous in capacity and move between shelters under constant speed, (4) adjourned passengers leave the line and will not be considered in the next period, (5) required trip time between two consecutive shelters refers to one time-slot, (6) buses dispatched at the same time-slot will bound for the same shelter, (7) passengers flow is only considered in one direction, (8) head-time between uses dispatched at the same time-slot is ignored, and (9) each bus may operate more than one loop a day.

2.2. Parameters and variables

To facilitate our analysis, we define following parameters and indices. We define by $K(i)$ the capacity of bus departed at time-slot $i$ (in person), by $\bar{D}(i)$ the operational cost (in rupiah per kilometer), by $D(i)$ the elapsed distance by bus from initial shelter in time-slot $i$ (in kilometer) and by $B$ the number of available buses in a corridor (in unit). We denote by index $i$ the $i$th time-slot ($i = 1, 2, ..., M$), by $j$ the $j$th departure shelter ($j = 1, 2, ..., N - 1$) and by $k$ the $k$th destination shelter ($k = 2, 3, ..., N$), from which we assume that there are $M$ time-slots and $N$ shelters along the corridor. Note that $i = 1$ refers to the first shelter where buses initially dispatched. Without loss of generality, we may assume that we have more time-slots than shelters, i.e., $M \geq N$.

We introduce the following decision variables. $K(i, j)$ denotes the total capacity of buses departed from shelter $j$ at time-slot $i$, $N_B(i, j)$ denotes the number of buses operated at shelter $j$ at time-slot $i$, $P(i, j)$ denotes the number of passengers supposed to be departed from shelter $j$ at time-slot $i$, $P^w(i, j, k)$ represents the number of lining-up passengers from shelter $j$ to shelter $k$ at time-slot $i$, $P^w(i, j)$ represents the total number of lining-up passengers at shelter $j$ at time-slot $i$, $P^{on}(i, j)$ and $P^{off}(i, j)$, respectively, denote the total number of getting-on and getting-off passengers at shelter $j$ at time-slot $i$, $P^b(i, j)$ represents the total number of adjourned passengers at shelter $j$ at time-slot $i$, $P^{ob}(i, j)$ denotes the total number of on-board passengers at shelter $j$ at time-slot $i$, $S(i, j)$ refers to the total number of available seats at shelter $j$ at time-slot $i$ just before passengers get-on the bus, $S^g(i, j)$ refers to the total number of available seats after embarkment and disembarkment at shelter $j$ at time-slot $i$, and $U(i, j)$ quantifies the utility level of bus departed from shelter $j$ at time-slot $i$. In this case, the utility value is calculated by the ratio between the total number of on-board passengers and the total capacity of buses.

2.3. Programming

In this part, we formulate the bus dispatching problem as an integer programming model. The objective of the model is to minimize operational cost expended by bus company throughout the period. The operational cost can be minimized by adjusting the number of dispatched buses at initial shelter. Thus, the objective function of the problem is given by

$$\min z := \bar{C} \sum_{i=1}^{M} \bar{D}(i) N_B(i, 1).$$
Subsequently, constraints involve in this problem mainly imposed by the transport demand between origin-destination points, infrastructure and operational instrument availability, and those related to regulation and standard.

1. The total number of lining-up passengers at shelter $j$ at time-slot $i$ is the whole lining-up passengers with various destinations:

$$P^w(i, 1) = \sum_{k=2}^{N} P(i, 1, k), \quad i = 1, 2, ..., M.$$  (2)

$$P^w(i, j) = \sum_{k=2}^{N} P(i, j, k), \quad i = 1, 2, ..., M, j \leq i.$$  (3)

2. The total number of getting-off passengers at shelter $k$ at time-slot $i$ is equal to those departed from shelter $j$ to shelter $k$ at preceding time-slots:

$$poff(i, k) = \sum_{j=1}^{k-1} P(M-(k-j), j, k), \quad i = 1, 2, ..., M, k \leq i.$$  (4)

From constraint (4) we obtain, for instance,

$$poff(5, 4) = P(2, 1, 4) + P(3, 2, 4) + P(4, 3, 4).$$

3. The number of passengers supposed to be departed at certain shelter and time-slot is equal to summation of that at preceding shelter and time-slot and the difference between lining-up and getting-off passengers, i.e., for $i = 1, 2, ..., M, j = 1, 2, ..., N-1$, and $j \leq i$, we have

$$P(i, j) = P(i-1, j-1, k) + P^w(i, j) - poff(i, j).$$  (5)

In case of first shelter and first time-slot, we have $P(i, 1) = P^w(i, 1)$ and $P(1, j) = P^w(1, j)$. Note that in (5), assumption on the correspondence between one time-slot and trip time between two shelters applies.

4. The total capacity of dispatched buses each time-slot should be greater than eighty percent of the number of passengers supposed to be departed from first shelter each time-slot:

$$N_R(i, 1)K(i) \geq 0.8 \max_{j \in M-(i-1)} P(j, j), \quad i = 1, 2, ..., M.$$  (6)

5. The total capacity of buses departed from shelter $j$ at time-slot $i$ is equal to the multiplication between the number of dispatched buses at first shelter and bus capacity:

$$K(i, j) = N_R(i, 1)K(i), \quad i = 1, 2, ..., M-1, j \leq i.$$  (7)

6. Constraints (8)-(15) relate to the number of getting-on passengers departed from certain shelter and time-slot.

- If the number of lining-up passengers at shelter 1 and time-slot $i$ is greater than or equal to its capacity, then the number of getting-on passengers is the same as capacity. And if smaller, all lining-up passengers will get-on the buses. Thus, for $i = 1, 2, ..., M$,

$$P^w(i, 1) \geq K(i, 1) \Rightarrow poff(i, 1) = K(i, 1).$$  (8)

$$P^w(i, 1) < K(i, 1) \Rightarrow poff(i, 1) = P^w(i, 1).$$  (9)

- The total number of available seats at shelter $i$ at time-slot $i$ just before passengers get-on the bus is equal to the capacity of the bus:

$$S(i, 1) = K(i, 1), \quad i = 1, 2, ..., M.$$  (10)

For the next shelters, the number of available seats is affected by the total number of on-board passengers as well as that of getting-off passengers. Thus,

$$S(i, j) = K(i, j) - poff(i-1, j-1) + poff(i, j), \quad i = 2, ..., M, j \leq i.$$  (11)
We also have the following conditional constraints for \( i = 1, 2, \ldots, M, j = 1, 2, \ldots, N - 1, \) and \( j \leq i \):

\[
S(i, j) \geq K(i, j) \rightarrow S(i, j) = K(i, j), \quad (12)
\]
\[
S(i, j) < K(i, j) \rightarrow S(i, j) = S(i, j), \quad (13)
\]
\[
S(i, j) \geq P^w(i, j) \rightarrow P^o(i, j) = P^w(i, j), \quad (14)
\]
\[
S(i, j) < P^w(i, j) \rightarrow P^o(i, j) = S(i, j). \quad (15)
\]

7. Constraints below relate to the total number of on-board passengers.

At the first shelter, this number is identical to that of getting-on passengers. While for the next shelters, it may be influenced by the number of getting-off passengers. Therefore, we possess

\[
p^o(i, 1) = p^o(i, 1), \quad i = 1, 2, \ldots, M, \quad (16)
\]
\[
p^o(i, j) = p^o(i - 1, j - 1) - p^{off}(i, j) + p^w(i, 1), \quad (17)
\]

Constraint (17) should be considered for \( i = 2, \ldots, M, j = 2, \ldots, N - 1, \) and \( j \leq i \). The following conditional constraints should also be applied:

\[
p^o(i, j) \leq 0 \rightarrow p^o(i, j) = 0, \quad j \leq i, \quad (18)
\]
\[
p^o(i, j) > 0 \rightarrow p^o(i, j) = p^o(i, j), \quad j \leq i. \quad (19)
\]

8. Next we must satisfy the following constraints in order to quantify the remaining available seats after embarkment and disembarkment of passengers at certain shelter and time-slot.

- The total number of available seats after embarkment and disembarkment at shelter \( j \) at time-slot \( i \) is equal to the difference between the total number of available seats just before passengers get-on the bus and that of getting-on passengers, i.e.,

\[
S_0(i, j) = S(i, j) - p^o(i, j), \quad j \leq i. \quad (20)
\]

9. The total number of adjourned passengers at shelter \( j \) and time-slot \( i \) is equal to the difference between the total number of lining-up passengers and that of getting-on passengers:

\[
P^a(i, j) = P^w(i, j) - p^o(i, j), \quad j \leq i. \quad (23)
\]

10. We need the following constraints to assure the trip continuity of each bus:

\[
N_B(i, 1) = N_B(i, i), \quad i = 2, \ldots, N, \quad (24)
\]
\[
N_B(i, 1) = N_B(i + j - 1, j), \quad i = 2, \ldots, M, j = 2, \ldots, N; \quad (25)
\]

where \( N_i \) denotes the index of last shelter to be bounded for when a bus is departed at time-slot \( i \). From (25), we may have, for instance,

\[
N_B(2, 1) = N_B(3, 2) = \ldots = N_B(3 + N_2, N_2).
\]

11. Utility level of buses at shelter \( j \) at time-slot \( i \) is defined by the ratio between the total number of on-board passengers and the total capacity of buses, i.e.,

\[
U(i, j) = \frac{p^o(i, j)}{K(i, j)}, \quad j \leq i. \quad (26)
\]

12. The total number of buses operated throughout the period does not exceed the number of available buses in a corridor:

\[
\sum_{i=1}^{M} N_B(i, 1) \leq \bar{N}. \quad (27)
\]

13. Integer constraint: \( N_B(i, j) \) are integers for all \( i \) and \( j \).
14. Non-negativity constraints: $P(i, j)$, $P^w(i, j)$, $P^{off}(i, j)$, $P^{on}(i, j)$, $p^{ob}(i, j)$, $N_g(i, j)$, $K(i, j)$, $S(i, j)$, $S_0(i, j)$ and $U(i, j)$ are non-negative for all $i$ and $j$.

3. Transjakarta Case

To illustrate the feasibility of the model, we consider a buses dispatching problem of Transjakarta transport system, also known as busway, a BRT system introduced by the Government of Jakarta. Starting with one corridor in 2004, currently Transjakarta manages twelve corridors consisting of more than 200 shelters. The system covers about 200 kilometers length, served by more than 600 units of bus. On average, Transjakarta delivers more than 350 thousands passengers a day.

To reduce the complexity of the problem, we applied the model only to Corridor I, which consists of 20 shelters connecting Blok M and Kota. Distance covered by this corridor is 13.8 kilometers and initially served by sufficient number of buses with uniform capacity 85 passengers. We here also limit the time horizon within one session which consists of 23 time-slots. The list of shelters in the corridor, their cumulative distances and average number of passengers in one direction (Blok M to Kota) are given in Table 1. The average number of passengers in a day at certain shelter is the summation of the number of passengers departed from this shelter to various destinations. As an example, Table 2 shows the number of passengers departed from third shelter Bundaran Senayan to other shelters in a session. Buses dispatched in the same time-slot have the same destination or route, while for different time-slot, it may differ. In the corridor, final stop for time-slot 1-5 is Kota, and subsequently Glodok, Olimo, Mangga Besar, Sawah Besar, Harmoni, Monas, Bank Indonesia, Sarinah, Bundaran HI, Tosari, Dukuh Atas, Setiabudi, Karet, Bendungan Hilir, Polda Metro Jaya, GBK, Bundaran Senayan and Al-Azhar. The determination of final stops in this work is merely affected by the termination of the session, i.e., up to time-slot 23. Normally the final stop for all buses is Kota, but in this analysis we did not consider any activities beyond time-slot 23. That is way the trip in the last time-slot just connects two consecutive shelters, e.g., Blok M to Al-Azhar or Bundaran Senayan to GBK as indicated by last row of Table 2. We further assume that the operational unit cost is 10435 rupiah per kilometer. Data of passengers is for 2011 and obtained from The Management of Transjakarta (UPTB, Unit Pengelola Transjakarta Busway). We aimed to determine the number of dispatched buses at each time-slot which minimized the total operational cost. We then compared our result which obtained by using operation research/management science (OR/MS) approach with that accomplished by UPTB.

<table>
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<tr>
<td>1</td>
<td>Blok M</td>
<td>0.00</td>
<td>5762</td>
</tr>
<tr>
<td>2</td>
<td>Al-Azhar</td>
<td>1.39</td>
<td>1022</td>
</tr>
<tr>
<td>3</td>
<td>Bundaran Senayan</td>
<td>2.12</td>
<td>1501</td>
</tr>
<tr>
<td>4</td>
<td>GBK</td>
<td>3.67</td>
<td>823</td>
</tr>
<tr>
<td>5</td>
<td>Polda Metro Jaya</td>
<td>4.18</td>
<td>854</td>
</tr>
<tr>
<td>6</td>
<td>Bendungan Hilir</td>
<td>4.98</td>
<td>1434</td>
</tr>
<tr>
<td>7</td>
<td>Karet</td>
<td>5.43</td>
<td>1067</td>
</tr>
<tr>
<td>8</td>
<td>Setiabudi</td>
<td>6.01</td>
<td>594</td>
</tr>
<tr>
<td>9</td>
<td>Dukuh Atas</td>
<td>6.45</td>
<td>420</td>
</tr>
<tr>
<td>10</td>
<td>Tosari</td>
<td>6.89</td>
<td>514</td>
</tr>
<tr>
<td>11</td>
<td>Bundaran HI</td>
<td>7.48</td>
<td>909</td>
</tr>
<tr>
<td>12</td>
<td>Sarinah</td>
<td>8.11</td>
<td>816</td>
</tr>
<tr>
<td>13</td>
<td>Bank Indonesia</td>
<td>8.70</td>
<td>343</td>
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<td>14</td>
<td>Monas</td>
<td>9.43</td>
<td>387</td>
</tr>
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<td>15</td>
<td>Harmoni</td>
<td>10.53</td>
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<tr>
<td>16</td>
<td>Sawah Besar</td>
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<td>17</td>
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<td>18</td>
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<td>19</td>
<td>Glodok</td>
<td>12.60</td>
<td>161</td>
</tr>
<tr>
<td>20</td>
<td>Kota</td>
<td>13.80</td>
<td>0</td>
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Table 3 describes the minimum number of dispatched buses and the number of passengers. It is shown that while UPTB dispatched 265 trips per session to transport passengers, calculation based OR/MS suggests a less number, it is only 100 trips required. By multiplying the number of dispatched buses, distance coverage, and operational unit cost according to (1) we corroborate that the total cost is 10366129 rupiahs. Obviously this is a 60 percent cost reduction. However, the consequence of dispatching less number of trips is that not all lining-up passengers could be departed, i.e., there were 1009 adjourned passengers (5.6 percent). This would not be a case of UPTB which decided to depart more trips. The numbers of getting-on and adjourned passengers presented in the table were acquired from passengers flow, as for the case of departure at time-slot 2 is depicted by Table 4. We can inspect that Table 4 accounts all the numbers affected by flow of passengers time by time. It is added up that the total numbers of getting-on and adjourned passengers are 1164 and 51, respectively, as summarized in Table 3. Utility value of 75 percent is come out by averaging utility values performed by buses in every shelter given in the last column of Table 4.
We have developed a simple deterministic buses dispatching problem with the main objective to minimize the number of departed buses from initial shelter each period. The state equations of the model were built based on the flow of lining-up, getting-on, getting-off and adjourned passengers. In the case of Transjakarta transport system, we have demonstrated that OR/MS approach elaborated in this paper can significantly reduce the number of dispatched buses. Extension can be made by relaxing assumption.
For example, it is realized that assumption 5 is too restrictive. Losing this assumption may expose the stochastic property of the model. In this case, for instance, trip time between shelters is a random variable and passengers arrival should be considered according to Poisson process. Readers may follow [6] for the direction.

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