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ESTIMATING THE INTENSITY OBTAINED AS THE PRODUCT OF A PERIODIC FUNCTION WITH THE QUADRATIC TREND OF A NON-HOMOGENEOUS POISSON PROCESS

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Abstract

A kernel-type nonparametric estimator of the intensity obtained as the product of a periodic function with the quadratic trend of a non-homogeneous Poisson process is constructed and investigated. It is considered the case when there is only a single realization of the Poisson process is observed in a bounded interval. The proposed estimator is proved to be weakly and strongly consistent when the size of the interval indefinitely expands. The asymptotic bias, variance, and the mean-squared error of the proposed estimator are also computed.

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1. Introduction

We consider a non-homogeneous Poisson process N on $[0, \infty)$ with ınknown) locally integrable intensity function λ. The intensity function λ is ;sumed to be a product of a periodic function with the quadratic trend. That , for any given point $s \in [0, \infty)$, the intensity function λ can be written as

$$\lambda(s) = (\lambda_c^*(s))(as^2), \tag{1.1}$$

ıere $\lambda_c^*(s)$ is a periodic function with known period τ and a denotes the ɔpe of the quadratic trend. We do not assume any (parametric) form of λ_c^* cept that it is periodic.

Since $a\lambda_c^*$ is also a periodic function with period τ, without loss of nerality, the intensity function λ given in (1.1) can also be written as

$$\lambda(s) = (\lambda_c(s))(s^2), \tag{1.2}$$

ere $\lambda_c(s) = a\lambda_c^*(s)$. Hence, for each point $s \in [0, \infty)$ and all $k \in \mathbf{Z}$, h $\mathbf{Z}$ denotes the set of integers, we have

$$\lambda_c(s + k\tau) = \lambda_c(s). \tag{1.3}$$

(1.2) and (1.3), the problem of estimating λ at a given point $s \in [0, \infty)$ be reduced to a problem of estimating λ_c at a given point $s \in [0, \tau)$. ıce, for the rest of the paper, we restrict our attention to the problem of mating λ_c at a given point $s \in [0, \tau)$.

Suppose that, for some $\omega \in \Omega$, a single realization $N(\omega)$ of the Poisson :ess N defined on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ with intensity function λ :n by (1.2) is observed, though only within a bounded interval $[0, n]$. Our in this paper is: (a) To construct a kernel-type estimator for λ_c at a :n point $s \in [0, \tau)$ using only a single realization $N(\omega)$ of the Poisson

process N observed in interval $[0, n]$. (b) To prove, under the minimal conditions, that our estimator is weakly and strongly consistent in estimating $\lambda_c(s)$. (c) To compute the asymptotic bias, variance, and the mean-squared error of the proposed estimator.

We will assume throughout that s is a Lebesgue point of λ, that is, we have

$$\lim_{h \downarrow 0} \frac{1}{2h} \int_{-h}^{h} |\lambda(s + x) - \lambda(s)| dx = 0 \tag{1.4}$$

(e.g., see [5, pp. 107-108]), which automatically means that s is a Lebesgue point of λ_c as well. This assumption is a mild one since the set of all Lebesgue points of λ is dense in $\mathbf{R}$, whenever λ is assumed to be locally integrable.

We refer to [4] for some parallel results when the intensity function λ is assumed to satisfy $\lambda(s) = (\lambda_c(s))(as)$, that is the product of a periodic function with the linear trend. See also [1] and [3] for some results when the intensity function is assumed to satisfy $\lambda(s) = \lambda_c(s) + as$.

2. Construction of the Estimator and Results

Let $K : \mathbf{R} \to \mathbf{R}$ be a real valued function, called *kernel*, which satisfies the following conditions: (K1) K is a probability density function, (K2) K is bounded, and (K3) K has (closed) support $[-1, 1]$. Let also h_n be a sequence of positive real numbers converging to 0, that is,

$$h_n \downarrow 0, \tag{2.1}$$

as $n \to \infty$. Now we may define the estimator of λ_c at a given point $s \in [0, \tau)$ as follows:

$$\hat{\lambda}_{c,n,K}(s) = \frac{\tau}{n} \sum_{k=0}^{\infty} \frac{1}{h_n (s + k\tau)^2} \int_0^n K\left(\frac{x - (s + k\tau)}{h_n}\right) N(dx). \tag{2.2}$$

Next we describe the idea behind the construction of the kernel-type imator $\hat{\lambda}_{c,n,K}(s)$ of $\lambda_c(s)$. Since there is available only one realization the Poisson process N, we have to collect necessary information about the ıknown) value of $\lambda_c(s)$ from different places of the interval $[0, n]$. For s reason, assumption (1.3) plays a crucial role and leads to the following ing of (approximate) equations. Let

$$N_n = \#\{k : s + k\tau \in [0, n]\},$$

ıere # denotes the number of elements. Then we have

$$\lambda_c(s) = \frac{1}{N_n}\sum_{k=0}^{\infty}\lambda_c(s+k\tau)\mathbf{I}\{s+k\tau \in [0, n]\}$$

$$= \frac{1}{N_n}\sum_{k=0}^{\infty}\frac{\lambda(s+k\tau)}{(s+k\tau)^2}\mathbf{I}\{s+k\tau \in [0, n]\}$$

$$\approx \frac{1}{N_n}\sum_{k=0}^{\infty}\frac{1}{(s+k\tau)^2}\frac{1}{2h_n}\int_{s+k\tau-h_n}^{s+k\tau+h_n}\lambda(x)\mathbf{I}(x \in [0, n])dx$$

$$= \frac{1}{N_n}\sum_{k=0}^{\infty}\frac{1}{2h_n(s+k\tau)^2}\mathbf{E}N([s+k\tau-h_n, s+k\tau+h_n]\cap[0, n])$$

$$\approx \frac{1}{N_n}\sum_{k=0}^{\infty}\frac{1}{2h_n(s+k\tau)^2}N([s+k\tau-h_n, s+k\tau+h_n]\cap[0, n])$$

$$\approx \frac{\tau}{n}\sum_{k=0}^{\infty}\frac{1}{2h_n(s+k\tau)^2}N([s+k\tau-h_n, s+k\tau+h_n]\cap[0, n]), \quad (2.3)$$

here $\mathbf{I}$ denotes the indicator function. We note that in order to make the first in (2.3) works, we have assumed that s is a Lebesgue point of λ and h_n ınverges to 0. Thus, from (2.3), we conclude that

$$\hat{\lambda}_{c,n}(s) = \frac{\tau}{n}\sum_{k=0}^{\infty}\frac{1}{2h_n(s+k\tau)^2}N([s+k\tau-h_n, s+k\tau+h_n]\cap[0, n]), \quad (2.4)$$

is an estimator of $\lambda_c(s)$. Note that the estimator $\hat{\lambda}_{c,n}(s)$ can be rewritten as

$$\hat{\lambda}_{c,n}(s) = \frac{\tau}{n}\sum_{k=0}^{\infty}\frac{1}{h_n(s+k\tau)^2}$$

$$\times \int_0^n \frac{1}{2}\mathbf{I}_{[-1,1]}([s+k\tau-h_n, s+k\tau+h_n])N(dx). \quad (2.5)$$

By replacing the function $\frac{1}{2}\mathbf{I}_{[-1,1]}(\cdot)$ in (2.5) by the general kernel $K(\cdot)$, we immediately arrive at the estimator introduced in (2.2).

Our main results are presented in the following theorems.

Theorem 1 (Consistency). *Suppose that the intensity function λ satisfies* (1.2) *and is locally integrable. If the kernel K satisfies conditions* (K1), (K2), (K3), *the bandwidth h_n satisfies assumptions* (2.1) *and $n^2h_n \to \infty$, then*

$$\hat{\lambda}_{c,n,K}(s) \xrightarrow{p} \lambda_c(s), \quad (2.6)$$

as $n \to \infty$, provided s is a Lebesgue point of λ. In other words, $\hat{\lambda}_{c,n,K}(s)$ is a consistent estimator of $\lambda_c(s)$. In addition, the Mean-Squared Error (MSE) of $\hat{\lambda}_{c,n,K}(s)$ converges to 0, *as $n \to \infty$, that is, we have*

$$MSE(\hat{\lambda}_{c,n,K}(s)) \to 0, \quad (2.7)$$

as $n \to \infty$.

Under naturally a stronger assumption on the choice of bandwidth h_n, we also have complete convergence of $\hat{\lambda}_{c,n,K}(s)$, which is given in the following theorem.

Theorem 2 (Complete convergence). *Suppose that the intensity function λ satisfies* (1.2) *and is locally integrable. If the kernel K satisfies conditions* (K1), (K2), (K3), *the bandwidth $h_n = n^{-\alpha}$ with $0 < \alpha < 1$, then*

$$\hat{\lambda}_{c,n,K}(s) \xrightarrow{c} \lambda_c(s),$$

as $n \to \infty$, provided s is a Lebesgue point of λ. In other words, $\hat{\lambda}_{c,n,K}(s)$ converges completely to $\lambda_c(s)$ as $n \to \infty$.

Note also that, by Theorem 2 and the Borel-Cantelli Lemma, we have strong consistency of $\hat{\lambda}_{c,n,K}(s)$, that is, we have

$$\hat{\lambda}_{c,n,K}(s) \xrightarrow{a.s.} \lambda_c(s),$$

as $n \to \infty$.

Asymptotic approximations to the bias and variance of $\hat{\lambda}_{c,n,K}(s)$ are given in the following two theorems.

Theorem 3 (Asymptotic approximation to the bias). *Suppose that the intensity function λ satisfies* (1.2), *is locally integrable and λ_c has finite second derivative λ_c'' at s. If the kernel K is symmetric and satisfies conditions* (K1), (K2), (K3), *h_n satisfies assumptions* (2.1) *and $nh_n^2 \to \infty$, then*

$$\mathbf{E}\hat{\lambda}_{c,n,K}(s) = \lambda_c(s) + \frac{1}{2}\lambda_c''(s)h_n^2 \int_{-1}^{1} x^2K(x)dx + o(h_n^2), \qquad (2.8)$$

as $n \to \infty$.

Theorem 4 (Asymptotic approximation to the variance). *Suppose that the intensity function λ satisfies* (1.2) *and is locally integrable. If the kernel K satisfies conditions* (K1), (K2), (K3) *and h_n satisfies assumptions* (2.1), *then*

$$Var(\hat{\lambda}_{c,n,K}(s)) = \frac{\pi^2\lambda(s)}{6n^2h_n}\int_{-1}^{1} K^2(x)dx + o\left(\frac{1}{n^2h_n}\right) \qquad (2.9)$$

as $n \to \infty$, provided s is a Lebesgue point of λ.

We note in passing that, the r.h.s. of (2.8) is the same as the r.h.s. of (2.9) of [4], the case when the intensity function is assumed to be the product of a periodic function with the linear trend. The same asymptotic approximation also appears in (2.5) of [2], the case when the intensity function is assumed to be purely periodic. However, we have slightly different asymptotic approximation to the variance of the estimator (cf. Theorem 4) compared to the one in [4] (cf. (2.10) of [4]) as well as the one in [2] (cf. (3.4) of [2]).

From Theorems 3 and 4, one can obtain an asymptotic approximation to the *Mean-Squared Error* (MSE) of $\hat{\lambda}_{c,n,K}(s)$, which is given by

$$MSE(\hat{\lambda}_{c,n,K}(s)) = \frac{\pi^2\lambda(s)}{6n^2h_n}\int_{-1}^{1} K^2(x)dx + \frac{1}{4}\left(\lambda_c''(s)\int_{-1}^{1} x^2K(x)dx\right)^2 h_n^4$$

$$+ o\left(\frac{1}{n^2h_n}\right) + o(h_n^4), \qquad (2.10)$$

as $n \to \infty$. By minimizing the sum of the first and second terms of (2.10) (the main terms for the variance and the squared bias), one can obtain the optimal choice of h_n, which is given by

$$h_n = \left[\frac{\pi^2\lambda(s)\int_{-1}^{1} K^2(x)dx}{\left(\lambda_c''(s)\int_{-1}^{1} x^2K(x)dx\right)^2}\right]^{\frac{1}{5}} n^{-\frac{2}{5}}.$$

With this choice of h_n, the optimal rate of decrease of $MSE(\hat{\lambda}_{c,n,K}(s))$ is of order $\mathcal{O}(n^{-8/5})$ as $n \to \infty$.

3. Proofs

Before proving Theorems 1, 2, 3 and 4, first we state and prove the following lemma, which is needed for proving Theorems 1 and 2.

Lemma 1 (Asymptotic unbiasedness). *Suppose that the intensity function* λ *satisfies* (1.2) *and is locally integrable. If the kernel* K *satisfies conditions* (K1), (K2), (K3) *and* h_n *satisfies assumptions* (2.1), *then*

$$\mathbf{E}\hat{\lambda}_{c,n,K}(s) \to \lambda_c(s), \tag{3.1}$$

as $n \to \infty$, *provided* s *is a Lebesgue point of* λ.

Proof. The expectation on the l.h.s. of (3.1) can be computed as follows:

$$\mathbf{E}\hat{\lambda}_{c,n,K}(s)$$

$$= \frac{\tau}{n}\sum_{k=0}^{\infty}\frac{1}{h_n(s+k\tau)^2}\int_0^n K\left(\frac{x-(s+k\tau)}{h_n}\right)\mathbf{E}N(dx)$$

$$= \frac{\tau}{n}\sum_{k=0}^{\infty}\frac{1}{h_n(s+k\tau)^2}\int_{\mathbf{R}} K\left(\frac{x-(s+k\tau)}{h_n}\right)\lambda(x)\mathbf{I}(x\in[0,n])dx. \tag{3.2}$$

By a change of variable and using (1.2) and (1.3), the r.h.s. of (3.2) can be written as

$$\frac{\tau}{n}\sum_{k=0}^{\infty}\frac{1}{h_n(s+k\tau)^2}\int_{\mathbf{R}} K\left(\frac{x}{h_n}\right)\lambda(x+s+k\tau)\mathbf{I}(x+s+k\tau\in[0,n])dx$$

$$= \frac{\tau}{nh_n}\int_{\mathbf{R}} K\left(\frac{x}{h_n}\right)\lambda_c(x+s)\sum_{k=0}^{\infty}\frac{(x+s+k\tau)^2}{(s+k\tau)^2}\mathbf{I}(x+s+k\tau\in[0,n])dx. \tag{3.3}$$

By noting that

$$\sum_{k=0}^{\infty}\frac{(x+s+k\tau)^2}{(s+k\tau)^2}\mathbf{I}(x+s+k\tau\in[0,n]) = \frac{n}{\tau}+O(1), \tag{3.4}$$

as $n \to \infty$ uniformly in $x \in [-h_n, h_n]$, we obtain

$$\mathbf{E}\hat{\lambda}_{c,n,K}(s) = \frac{1}{h_n}\int_{\mathbf{R}} K\left(\frac{x}{h_n}\right)\lambda_c(x+s)dx + O\left(\frac{1}{n}\right), \tag{3.5}$$

as $n \to \infty$. The first term on the r.h.s. of (3.5) can be written as

$$\frac{1}{h_n}\int_{\mathbf{R}} K\left(\frac{x}{h_n}\right)(\lambda_c(x+s)-\lambda_c(s))dx + \frac{\lambda_c(s)}{h_n}\int_{\mathbf{R}} K\left(\frac{x}{h_n}\right)dx. \tag{3.6}$$

Since s is a Lebesgue point of λ_c (cf. (1.4)) and the kernel K satisfies conditions (K2) and (K3), it is easily seen that the first term of (3.6) is $o(1)$, as $n \to \infty$. By the assumption $\int_{\mathbf{R}} K(x)dx = 1$ (cf. (K1)), the second term of (3.6) is equal to $\lambda_c(s)$. Clearly, the second term on the r.h.s. of (3.5) is $o(1)$, as $n \to \infty$. Hence, the r.h.s. of (3.5) is equal to $\lambda_c(s)+o(1)$, as $n \to \infty$. This completes the proof of Lemma 1.

Proof of Theorem 3. By (2.1) and Young's form of Taylor's theorem, the first term on the r.h.s. of (3.5) can be written as

$$\frac{1}{h_n}\int_{-h_n}^{h_n} K\left(\frac{x}{h_n}\right)\lambda_c(x+s)dx = \int_{-1}^{1} K(x)\lambda_c(s+xh_n)dx$$

$$= \lambda_c(s) + \lambda_c'(s)h_n\int_{-1}^{1} xK(x)dx + \frac{\lambda_c''(s)}{2}h_n^2\int_{-1}^{1} x^2K(x)dx + o(h_n^2),$$

as $n \to \infty$. Since the kernel K is symmetric around zero, we have that $\int_{-1}^{1} xK(x)dx = 0$. By the assumption $nh_n^2 \to \infty$, the second term on the r.h.s. of (3.5) is $o(h_n^2)$, as $n \to \infty$. Hence we have (2.8). This completes the proof of Theorem 3.

Proof of Theorem 4. The variance of $\hat{\lambda}_{c,n,K}(s)$ can be computed as follows:

$$Var(\hat{\lambda}_{c,n,K}(s)) = \frac{\tau^2}{n^2}Var\left(\sum_{k=0}^{\infty}\frac{1}{h_n(s+k\tau)^2}\int_0^n K\left(\frac{x-(s+k\tau)}{h_n}\right)N(dx)\right). \tag{3.7}$$

By (2.1), for sufficiently large n, we have that the intervals $[s + k\tau - h_n, s + k\tau + h_n]$ and $[s + j\tau - h_n, s + j\tau + h_n]$ are not overlap for all $k \neq j$. This implies, for all $k \neq j$,

$$K\left(\frac{x - (s + k\tau)}{h_n}\right)N(dx) \text{ and } K\left(\frac{x - (s + j\tau)}{h_n}\right)N(dx)$$

are independent random variables. Hence, the r.h.s. of (3.7) can be computed as follows:

$$\frac{\tau^2}{n^2h_n^2}\sum_{k=0}^{\infty}\frac{1}{(s + k\tau)^4}\int_0^n K^2\left(\frac{x - (s + k\tau)}{h_n}\right)Var(N(dx))$$

$$= \frac{\tau^2}{n^2h_n^2}\sum_{k=0}^{\infty}\frac{1}{(s + k\tau)^4}\int_0^n K^2\left(\frac{x - (s + k\tau)}{h_n}\right)\mathbf{E}N(dx)$$

$$= \frac{\tau^2}{n^2h_n^2}\sum_{k=0}^{\infty}\frac{1}{(s + k\tau)^4}\int_0^n K^2\left(\frac{x - (s + k\tau)}{h_n}\right)\lambda(x)dx. \tag{3.8}$$

By a change of variable and using (1.2) and (1.3), the r.h.s. of (3.8) can be written as

$$\frac{\tau^2}{n^2h_n^2}\sum_{k=0}^{\infty}\frac{1}{(s + k\tau)^4}\int_{\mathbf{R}} K^2\left(\frac{x}{h_n}\right)\lambda(x+s+k\tau)\mathbf{I}(x+s+k\tau \in [0, n])dx$$

$$= \frac{\tau^2}{n^2h_n^2}\int_{\mathbf{R}} K^2\left(\frac{x}{h_n}\right)\lambda_c(x+s)\sum_{k=0}^{\infty}\frac{(x+s+k\tau)^2}{(s+k\tau)^4}\mathbf{I}(x+s+k\tau \in [0, n])dx. \tag{3.9}$$

Now note that

$$\sum_{k=0}^{\infty}\frac{(x + s + k\tau)^2}{(s + k\tau)4}\mathbf{I}(x + s + k\tau \in [0, n]) = \frac{\pi^2}{6\tau^2} + o(1), \tag{3.10}$$

as $n \to \infty$ uniformly in $x \in [-h_n, h_n]$. Then, the r.h.s. of (3.9) is equal to

$$\frac{\pi^2}{6n^2h_n^2}\int_{\mathbf{R}} K^2\left(\frac{x}{h_n}\right)\lambda_c(x + s)dx + o\left(\frac{1}{n^2h_n}\right). \tag{3.11}$$

Since s is a Lebesgue point of λ_c (cf. (1.4)) and the kernel K has support in $[-1, 1]$, we see that the first term on the r.h.s. of (3.11) is equal to the r.h.s. of (2.9). We also see that the second term of (3.11) is of the same order as the second term on the r.h.s. of (2.9). This completes the proof of Theorem 4.

Proof of Theorem 1. By Lemma 1, Theorem 4 and the assumption $n^2h_n \to \infty$ as $n \to \infty$, we have (2.6) and (2.7). This completes the proof of Theorem 1.

Proof of Theorem 2. To prove Theorem 2, we have to show, for any $\varepsilon > 0$,

$$\sum_{n=1}^{\infty}\mathbf{P}(|\hat{\lambda}_{c,n,K}(s) - \lambda_c(s)| > \varepsilon) < \infty. \tag{3.12}$$

First note that

$$\mathbf{P}(|\hat{\lambda}_{c,n,K}(s) - \lambda_c(s)| > \varepsilon) \leq \mathbf{P}(|\hat{\lambda}_{c,n,K}(s) - \mathbf{E}\hat{\lambda}_{c,n,K}(s)| + |\mathbf{E}\hat{\lambda}_{c,n,K}(s) - \lambda_c(s)| > \varepsilon).$$

By Lemma 1, there exists a positive real number M such that, for all $n > M$, $|\mathbf{E}\hat{\lambda}_{c,n,K}(s) - \lambda_c(s)| < \varepsilon/2$. This implies, for sufficiently large n,

$$\mathbf{P}(|\hat{\lambda}_{c,n,K}(s) - \lambda_c(s)| > \varepsilon) \leq \mathbf{P}\left(|\hat{\lambda}_{c,n,K}(s) - \mathbf{E}\hat{\lambda}_{c,n,K}(s)| > \frac{\varepsilon}{2}\right)$$

$$\leq \frac{4Var(\hat{\lambda}_{c,n,K}(s))}{\varepsilon^2}, \tag{3.13}$$

by the Chebyshev inequality. From Theorem 4 and the assumption $h_n = n^{-\alpha}$ with $0 < \alpha < 1$, we see that the r.h.s. of (3.13) is $\mathcal{O}(n^{\alpha-2})$, which is summable, and Theorem 2 follows. This completes the proof of Theorem 2.

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