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POSTER

SPECTRAL APPROACH FOR TIME SERIES ANALYSIS

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1. Introduction

The first appearance of spectral analysis in the study of macroeconomic time series dates motivated by the requirement of a more insightful knowledge of the series structure and supported by the contemporaneous progress in spectral estimation and computation. The first works focused on the problem of seasonal adjustment procedures and on the general spectral structure of economic data. Cross spectral methods were pointed out from the outset as being important in discovering and interpreting the relationships between economic variables. After the early years, the range of application of such analysis was extended to the study of other econometric issues, among which the controversial trend-cycle separation, the related problem of business cycles extraction and the analysis of co-movements among series, usefiil in the study of international business cycles. In particular, cross spectral analysis allows a detailed study of the correlation among series. An empirical investigation about the possibility that the market is in a self-organized critical state (SOC) show a power law behaviour in the avalanche size, duration and laminar times during high activity period (Bartolozzi, Leinweber and Thomas, 2005).

2. Mathematical Models for Fractal Fluctuations and Deterministic

The larger scale fluctuations incorporate the smaller scale fluctuations as intemal fine scale structure and may be visualized as a continuum of eddies (waves). Dynamical systems in nature are basically fiizzy logic systems integrating a unified v^ole communicating network of input signals (perturbations) with self-organized ordered two-way information transpot between the larger and smaller scales. Self-similar structures are generated by iteration (repetition) of simple rules for growth processes on all scales of space and time. Such iterative processes are simulated mathematically by numerical computations such as *X„+i* = F(^«) where *X„+i* the value of the variable X at (« + *if"* computational step is a fiinction F of its earlier value X„ The spectrum of *{x}* is defined to be the Fourier transform of Yx(/^),

$$\frac{1}{2\pi}\sum_{h=-\infty}^{\infty} e^{-ih\omega}\gamma_x(h), \quad \gamma_x(h) = E(x_t x_{t-h})$$

3. Model Concepts of Universal Spectrum of Fluctuations

A general systems theory was first developed to quantify the observed fractal space-time fluctuations in turbulent fluid flows. The model concepts are independent of the exact details of the physical, chemical or other properties and therefore applicable to all dynamical systems. The observed inverse power law form for power spectra implies that the fi-actal fluctuations can be visualized to result from a hierarchical eddy continuum structure for the overall pattern. Starting from this simple basic concept that large scale eddies form as envelopes of intemal small scale eddy circulations the following important model predictions are derived.

Time-fi-equency approaches which represent the fi*equency content of a series, while keeping the time description parameter to give a three-dimensional time-dependent spectrum will not be tackled in this paper. This is for essentially two reasons: first, they would require more than a simple section; second, and more importantly, because evolutionary spectral methods and wavelets are suitable when dealing wdth very long time series, like those found in geophysics, astrophysics, neurosciences or finance. But their application to short series the norm in macroeconomics is difficult and may give unstable parameter-dependent results. For such series, traditional spectral analysis is probably more suitable.

Let Z_t is time series stationer process with autocorrelation γ_k. Its Fourier transformation is (Yujima, 1998) :

$$f(w) = \frac{1}{2p} \sum_{k=-\infty}^{\infty} g_k e^{-iwk}$$

Function *f(w)* is continu function non-negative that representative of spectrum for autocorrelation function γ_k. Spectrum of *f(w)* and autocorrelation function γ_k are unique Fourier transformation. Generally, spectrum of ARMA(p,q) process can be written (West, 1999):

$$f(w) = \frac{s_a^2 q_q(e^{-iw}) q_p(e^{iw})}{2p f_p(e^{-iw}) f_p(e^{iw})}$$

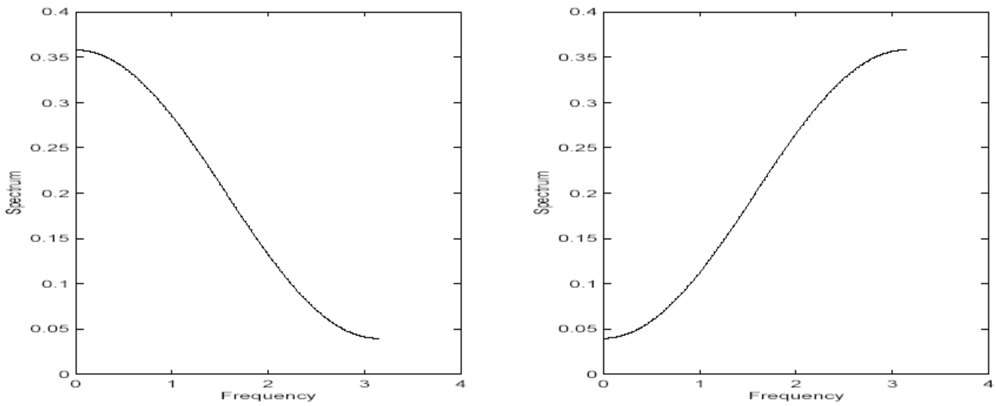

Figure 1: Plots of the spectrum of MA(1) processes (0 = 0.5 for the left figure and 0 = - 0.5 for the right figure) – Simulated Data

Figure 1 plots the spectrum of MA(1) processes with positive and negative coefficients. When 0 > 0, we see that the spectrum is high for low frequencies and low for high frequencies. When 0 < 0, we observe the opposite. This is because when 0 is positive, we have positive one lag correlation which makes the series smooth with only small contribution from high frequency (say, day to day) components. When 0 is negative, we have negative one lag correlation, therefore the series fluctuates rapidly about its mean value.

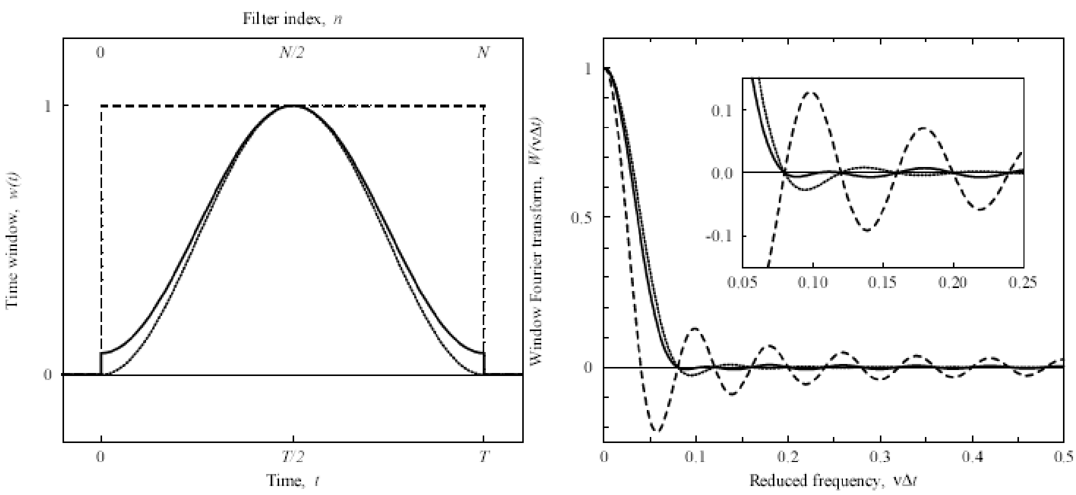

Figure 3: Window Functions and Their Frequency Response (Simulated Data). *The rectangular (dashedline), Hanning (dotted line) and Hamming (full line) time windows (left panel) and their*

respective Fourier transforms (right panel). For the latter, the number of points $N = 15$ has been chosen rather small to emphasize the differences. Note in the zoom (right panel, inset) the reduced side lobe amplitude and leakage of the Hanning and Hamming windows with respect to the rectangular one, the Hamming window performing better in the first side lobe.

The only way to prevent this effect, would be to choose T (or equivalently N) as a multiple of the largest period that is likely to occur. Unfortunately, this is feasible only if we have some idea of the frequencies involved in the process and would in any case entail some loss of data at one or both sample ends. As for the cutoff frequencies $\nu_l = k_l/(N\Delta t)$ and $\nu_h = k_h/(N\Delta t)$, given the value of N, they must be chosen to be multiples of T^{-1}, otherwise the filter does not completely remove the zero frequency component (i.e. the signal mean) and cannot help in eliminating unit roots.

4. Conclusion

This paper highlighted the main features of spectral analysis and their practical application. After a general theoretical introduction, we approached the issue of filtering for the extraction of particular components, mostly those related to the business cycle. In fact one of the advantages of the method is that it allows a quantitative definition of the cycle, and the extraction of long, medium or short term components, according to the researcher's wish. Then, we sketched the theory and practice of cross spectral analysis introducing some typical concepts, like coherency and phase spectrum, which may provide some essential information, complementary to that given by time domain methods.