PROCEEDINGS OF THE 3rd SEMINAR

TOWARD HARMONIZATION BETWEEN DEVELOPMENT AND ENVIRONMENTAL CONSERVATION IN BIOLOGICAL PRODUCTION

December, 5th - 2004
Serang, Banten (INDONESIA)

Organized Jointly by

Bogor Agricultural University
The University of Tokyo
Government of Banten Province
PT. Krakatau Tirta Industri
MODELING SOLUTE TRANSPORT IN SOILS WITH PITCHER FERTIGATION SYSTEM

HERMANTORO 1), Budi I. SETIAWAN 2), Sho SHIOZAWA 3)

1) Stiper Agricultural University, JOGJAKARTA
2) Bogor Agricultural University, BOGOR
3) University of Tokyo JAPAN

Abstract

There is high motivation to develop pitcher irrigation system due to its effectiveness in wetting the root zone. Many experiments have been conducted to study water flow in soils with the application of pitcher irrigation. The objective of this research is to develop models to study solute transport in the soils. Numerical approaches with 2-D finite difference method were applied and verified. The results show the models are more or less confirmed well with measured data.

Key words: Solute Distribution, Pitcher, Fertigation, Simulation.

INTRODUCTION

According to Feyen et. al. (1998) solute transport in porous media is determined by two parameters, i.e.: 1) average velocity of solute particle, and 2) solute dispersion. Solute dispersion effected by two entities, namely 1) molecular diffusion and 2) hydrodynamic dispersion. By using the continuum approach, convection equation - dispersion solute transport of non-adsorbing and non-degradable, shall be as follows:

\[ \frac{\partial c}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\partial c}{\partial x} \right) - \frac{\partial c}{\partial x} = \frac{D_{sh}}{\theta} \frac{\partial^2 c}{\partial x^2} \]

Where:
- \( \theta \) : volumetric water content (cm\(^3\)/cm\(^3\))
- \( C \) : solution concentration (gram/cm\(^3\))
- \( V \) : average velocity of solution (cm/\( \text{dt} \))
- \( X \) : direction of flow (cm)
- \( t \) : time increment (\( \text{dt} \))
- \( D_{sh} \) : hydrodynamic-dispersion coefficient

One dimension solute transport equation in soil with simple condition can be solved by analytical method. For two dimensions or more solute transport is generally finished by numerical method trough finite different method or finite element method.

Setiawan (1998) have made the model of water distribution from pitcher irrigation using two dimension finite different methods. The simulation model of water distribution in soil from pitcher irrigation system by finite element method has been made Saleh (2000). Base on this simulation we found that the distribution rate of irrigation water around the pitcher is influenced by interaction of pitcher and soil characteristic. Noborio et. al. (1996) built the two dimension transport model of water, heat, and solute at furrow irrigation system using finite element method.
The objective of the research is to develop the model and simulation of solute distribution in soil on pitcher fertigation system using the finite different method.

**MATERIAL AND METHOD**

Materials used for experiment are NaCl solution, distilled water, box with the front side made from transparent glass (10 x 75 x 100) cm³, cutting pitcher with the hydraulic conductivity 6.7 x 10⁻⁸ cm/sec and diffusivity coefficient of 1.1 x 10⁻⁸ cm²/sec, Leuwikopo soil with the saturated hydraulic conductivity 7.7 x 10⁻³ cm/sec diffusivity coefficient of 4.1 x 10⁻³ cm²/sec, Mariotte tube, plastic pipe, black silver plastic, stopwatch and Electrical conductivity meter.

Experiment of solute distribution of in soil is conducted in soil box. Pitcher cutting with its both sides is closed by an acrylic plate attached in the middle of soil box. Pitcher is supplied water and NaCl solution with the constant head from a mariotte tube. The water is supplied to the soil until reached the steady state flow. After reached steady state condition the water in pitcher swiftly changed with the Nacl solution which have been known its concentration. Concentration of NaCl in soil is estimated by electrical conductivity of soil sample at various distances around the pitcher.

The verification of the model of solute distribution in soil conducted experiment by using vertical flimsy cutting the pitcher. Base of experiment the solute transport equation in soil on pitcher fertigation system can eliminate as a two dimension solute transport as follows:

\[
\frac{\partial c}{\partial t} = D_{sh} \frac{\partial^2 c}{\partial x^2} + D_{sh} \frac{\partial^2 c}{\partial z^2} - v_x \frac{\partial c}{\partial x} - v_z \frac{\partial c}{\partial z} \tag{2}
\]

Assumptions used in solving the equation of solute transport model as follows: 1) Pitcher and soil are represent thehomogeneous porous media and isotropic, 2) water only seep from pitcher wall, part of base and pitcher neck made impervious. There are no water source except from within pitcher with constant head water surface. 3) Soil moisture of pitcher wall always in saturated condition, 4) Two dimension flow of solute are vertical (z) and horizontally direction (x), 5) The simulation of solute transport is done after steady state condition of water flow, with initial condition \(c(x,z,0) = 0\).

Boundary Condition to solve the solute transport simulation in two dimensions shall be as follows:

1. The soil surface is covered by the black silver plastic so the evaporation From soil surface is not happened.

\[
q_x(x,z,t) = -\frac{K(\theta)}{C_w(\Psi) \partial \theta} K(\theta) \partial \theta + K(\theta) = 0 \quad \text{With} \quad 0 \leq x \leq X, z = 0 \quad \text{and} \quad t > 0
\]

2. No water flow from the boundary condition of soil box.

\[
q_x(x,z,t) = 0 \quad \text{At} \quad x = X, 0 \leq z \leq Z \quad \text{and} \quad t > 0
\]

3. No percolation of water from base of soil box.

\[
q_x(x,z,t) = 0 \quad \text{With} \quad 0 \leq x \leq X, z = Z \quad \text{dan} \quad t > 0
\]

Pitcher wall is always in saturated condition, hence water flow at pitcher wall following the equation:
\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K_{\text{trend}} \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_{\text{trend}} \left(\frac{\partial h}{\partial z} + 1\right)\right)
\]  
(3)

While water flow in soil around the pitcher represent the unsaturated flow as follows:

\[
C \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial x}\left(K_{(\psi)} \frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_{(\psi)} \left(\frac{\partial \psi}{\partial z} + 1\right)\right)
\]  
(4)

where:

\[
C_{(\psi)} = \frac{\partial \theta}{\partial \psi}
\]

Is specific water capacity

\[\psi:\text{suction head (cm H2O)}\]
\[\theta:\text{Volumetric water content (cm3/cm3)}\]
\[k:\text{soil hydraulic conductivity (cm/sec)}\]
\[x:\text{horizontal direction (cm)}\]
\[z:\text{vertical direction (cm)}\]
\[t:\text{time (sec)}\]

Equation (3) and (4) is an identical form and can be solved together using the pitcher hydraulic conductivity \(K_{\text{trend}}\) when the solute transport passes the pitcher wall and the value soil hydraulic conductivity \(K_{(\psi)}\) if solute passing the soil. Unsaturated soil hydraulic conductivity calculated using equation which have been developed by Setiawan and Nakano (1993)

\[K(\theta) = K_s \times \text{EXP}(-a(\theta_s - \theta)^b)\]  
(5)

where:

\[K(\theta):\text{Unsaturated hydraulic conductivity as function of } \theta \text{ (cm/sec)}\]
\[K_s: \text{saturated hydraulic conductivity (cm/sec)}\]
\[\theta_s: \text{Volumetric water content saturated (cm3/cm3)}\]
\[\theta: \text{Volumetric water content (cm3/cm3)}\]

The value of parameters above for Leuwikopo soil type is a: 11.20 and b: 0.1354 (Saleh, 2000).

Soil water retention, \(\Psi\) as a function of water content \(\theta\) calculated using equation which developed by Van Genuchten (1980) after modified by Setiawan (1992) as follows:

\[
\theta = \theta_s + \frac{\theta_s - \theta_r}{\left[1 + \left(\frac{\text{abs}(\Psi - h_{\text{max}})}{\alpha}\right)\right]^n}\]
(6)

For Leuwikopo soil type the coefficient of \(\theta_s = 0.679; \theta_r = 0.201; \alpha = 69,835 \quad n = 2,743; \quad m = 1,196 \quad \text{and } h_{\text{max}} = 20 \) (Saleh, 2000).
The modeling and simulation of solute distribution in soil on pitcher fertigation developed using finite different method with Alternate Directing Implicit (ADI) scheme (Setiawan, 1992). In the ADI scheme, the system of non-linear equation transform to a linear equation with the Newton method, and then the linear equation system is solved simultaneously by Thomas algorithm. Verification the model is done by comparing concentration of potassium chloride (NaCl) at some point on certain time, C(x,z,t) between model output and measured/observed concentration. The method which selected is graphical analysis or simple regression and RMSE (Root Mean Square Error).

The water flow equation is solved in advance to get the velocity of water flow and soil moisture distribution in steady state condition. Later then we used the above parameter to solve the solute transport equation. The flow chart of finite different method is presented in figure 1.

Mathematically the algorithm of ADI scheme is as follows:

**Water flow**

\[
\begin{align*}
C_{i,j} \frac{\psi^{i+1/2}_{i,j} - \psi^i_{i,j}}{\Delta t} &= \left\{ \frac{k_{i+1,j} - k_{i-1,j}}{2\Delta x} \psi_{i+1,j}^{i+1/2} - \psi_{i,j}^i + k_{i,j} \psi_{i+1,j}^i - 2\psi_{i,j}^i + \psi_{i-1,j}^i \right\} + \\
&\left\{ \frac{k_{i+1,j} - k_{i,j}}{2\Delta x} \psi_{i+1,j+1}^{i+1/2} - \psi_{i,j+1}^i + k_{i,j} \psi_{i+1,j+1}^{i+1/2} - 2\psi_{i,j+1}^i + \psi_{i,j}^i \right\} \frac{\Delta x^2}{2\Delta z} \\
C_{i,j} \frac{\psi^{i+1/2}_{i,j} - \psi^i_{i,j}}{\Delta t} &= \left\{ a\psi_{i+1,j}^{i+1/2} - \psi_{i,j}^i + b\psi_{i+1,j}^{i+1/2} - 2\psi_{i,j}^i + \psi_{i-1,j}^i \right\} + \\
&\left\{ a\psi_{i,j+1}^{i+1/2} - \psi_{i,j}^i + b\psi_{i,j+1}^{i+1/2} - 2\psi_{i,j}^i + \psi_{i,j}^i \right\} \frac{\Delta x^2}{2\Delta z} \\
\end{align*}
\]

(7)

Where:

\[
ax = \frac{k_{i+1,j} - k_{i-1,j}}{2\Delta x}, \quad bx = k_{i,j}, \quad az = \frac{k_{i+1,j} - k_{i-1,j}}{2\Delta z}, \quad bz = k_{i,j}
\]

First stage we solve at x-direction (columns)

\[
fi = ai\left( \psi^{i+1/2}_{i-1,j} \right) + bi\left( \psi^{i+1/2}_{i,j} \right) + ci\left( \psi^{i+1/2}_{i+1,j} \right) - di = 0
\]

(9)

Where:

\[
ai = \frac{bx}{\Delta x^2} - \frac{ax}{2\Delta x}; \quad bi = -\frac{2bx}{\Delta x^2} - \frac{2bz}{\Delta z^2} + \frac{C}{\Delta t}; \\
\]

\[
ci = \frac{ax}{2\Delta x} + \frac{bx}{\Delta x^2}; \quad di = aj\psi_{i,j-1}^i + bj\psi_{i,j}^i + cj\psi_{i,j+1}^i - az
\]

The second stage ADI scheme is solves at z-direction

\[
fj = aj\left( \psi^{i+1/2}_{i,j-1} \right) + bj\left( \psi^{i+1/2}_{i,j} \right) + cj\left( \psi^{i+1/2}_{i,j+1} \right) + az - dj = 0
\]

(10)

Where:

\[
aj = \frac{bz}{\Delta z^2} - \frac{az}{2\Delta z}; \quad bj = -\frac{2bx}{\Delta x^2} - \frac{2bz}{\Delta z^2} + \frac{C}{\Delta t}; \\
\]

\[
cj = \frac{az}{2\Delta z} + \frac{bz}{\Delta z^2}; \quad dj = ai\psi_{i-1,j}^{i+1/2} + bi\psi_{i,j}^{i+1/2} + cj\psi_{i+1,j}^{i+1/2}
\]
Figure 1 Flow chart of two dimensional finite different for solute transport

Solute Transport

\[
\frac{\partial c}{\partial t} = -V_x \frac{\partial c}{\partial x} + \frac{D}{\theta} \frac{\partial^2 c}{\partial z^2} - V_z \frac{\partial c}{\partial z} + \frac{D}{\theta} \frac{\partial^2 c}{\partial z^2}
\]

(11)

\[
\frac{C_{i,j}^{t+\Delta t/2} - C_{i,j}^t}{\Delta t} = -V_{x_{i,j}} \frac{C_{i+1,j} - C_{i-1,j}}{2\Delta x} + \frac{D}{\theta_{i,j}} \left( \frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j}}{\Delta x^2} \right)
\]

\[
- V_{z_{i,j}} \frac{C_{i,j+1} - C_{i,j-1}}{2\Delta z} + \frac{D}{\theta_{i,j}} \left( \frac{C_{i,j+1} - 2C_{i,j} + C_{i,j-1}}{\Delta z^2} \right)
\]

(12)

Hereinafter solute transport in soil is solved identically with water flow equation.
RESULT AND DISCUSSIONS

1. Computer Simulation Programming

Modeling and simulation of solute distribution in soil on pitcher fertigation is programmed by Delphi 6 computer program language. This model consists of seven bottom groups, i.e.: 1) Soil characteristic, 2) soil box dimension, 3) Solute transport characteristic, 4) Time of simulation, 5) Saving simulation output, 6) output of simulation, and 7) Help, close, or execution menus. The visual form is presented Appendix 1.

2. Soil moisture distribution

The result of water flow simulation of fertigation system is expressed as soil moisture distribution around the pitcher that is presented at Figure 2. This figure shows that the water content at position progressively far from pitcher wall at horizontal and vertical direction is decrease gradually. This matter is the effect of gravitation in vertical direction, like at equation (3).

![Figure 2. Water content distribution or simulation in steady state condition.](image)

The volumetric water content distribution around of 0.67 at near of pitcher wall and about 0.25 at 21 cm in the horizontal distance from the pitcher wall and 17 cm under the bottom of the pitcher. Its not significant different if compared with the result of simulation by Setiawan (1998) using the pitcher hydraulic conductivity < 10 soil hydraulic conductivity.
Figure 3. The water content from simulation (prediction) versus observed

The reliability verification the simulation model obtained RMSE of 0.083 and
graphically presented at Figure 3. The regression between water content prediction
and measured data is \( y = 0.9136 \times x \), by \( R^2 = 0.81 \).

3. Distribution of Solute Concentration

The distribution of solute concentration as a function of time on certain point,
\( C(x,z,t) \) is presented in Figure 4.

From Picture 4 seen by that distribution of concentration of salt solution in soil
\( (\text{KCl}) \) around the pitcher in the reality is similar with water content distribution. This
water can be explain easily that dissolve salt mass in soil will make a move through
invection process follow the velocity of water flow and diffusion process, as its
result the salt concentration found just only just in wet soil. The concentration of salt
distribution at vertical direction reach 45 cm deepness and at horizontal distance
about 25 cm from pitcher wall with the concentration relative equal to 0,10.
Concentration at near the pitcher wall at 1 cm distance reaches 0.89 relative to
concentration of 1 g / l in pitcher.

This concentration distribution of NaCl is verified with the data from
experiment at several point of soil around the pitcher. The result of verification
indicate that the output concentration distribution from the simulation are non
significant different with the measurement data, that assign value RMSE equal to
0.1725. The simple regression equation between concentration prediction and
measurement data is \( y = 0.874 \times x \), by \( R^2 = 0.81 \) like presented at Figure 5.

![Figure 5. Concentration of NaCl solution predicted versus observed](image)

CONCLUSION

1. The solute concentration in soil around the pitcher has a decreasing trend by
distance increment from pitcher. The spreading of NaCl at horizontal direction
reaches the distance of about 25 cm and 45 cm at vertical direction with the
relative concentration of 0,1g/l.
2. Concentration distribution in soil around the pitcher is influenced by hydraulic
conductivity of pitcher and soil, water retention curve and diffusion coefficient.
3. The ADI two dimensional finite different method give the satisfying result in
modeling and simulation of solute distribution in soil on pitcher fertigation system.

REFERENCES

flow and solute transport in heterogeneous soils, A review of Recent
Noborio K., K. J. McInnes, and J. L. Heilman, 1996. Two-dimentional model for
Hermantoro (2003). Efektivitas Sistem Fertigasi Kendi Kasus pada Tanaman Lada
Perdu. Disertasi diajukan pada Program Studi Ilmu Keteknikan Pertanian
Fakultas Pasca Sarjana Institut Pertanian Bogor.


Appendix 1. Form of computer programming solute distribution in soil on pitcher fertgation system.