Penurunan sistem persamaan (18a-18c) dari sistem persamaan (17a-17c)

Penurunan sistem persamaan (18a-18c) didefinisikan variabel-variabel sebagai berikut:

\[ P = S + I \text{ dan } i = \frac{1}{P} \]

sama dengan

\[ \mathcal{I} = \frac{I}{P} \Rightarrow I = Pi \]

\[ P = S + I \Rightarrow S = P - I = P - Pi \]

Selanjutnya dilakukan substitusi ke dalam sistem persamaan (17a), (17b), dan (17c) untuk mendapatkan sistem persamaan (18a), (18b), dan (18c) secara berurutan.

Untuk persamaan (18a)

Persamaan (18a) diperoleh dengan menurunkan persamaan \( \mathcal{S} \) terhadap waktu \( t \) yaitu

\[
\frac{dP}{dt} = \frac{aS}{1+b(S+I)} Z - \gamma \frac{S}{S+I} m_i \]

\[
= r_i (P-iP)(1-(P-iP)-iP) - \frac{a(P-iP)}{1+bP} Z - \gamma \frac{(P-iP)pP}{P} \]

\[
= r_i iP(1-(P-iP) - iP) - \frac{a(P-iP)}{1+bP} Z - \gamma \frac{(P-iP)pP}{P} - m_i P
\]

\[
= -aP (1-i) + r_i iP(1-P) - \frac{aS}{1+bP} Z - \frac{aS}{1+bP} (S+I) - m_i P
\]

\[
= r_i (1-i) + r_i iP(1-P) - \frac{aS}{1+bP} Z - m_i P
\]

\[
\therefore \text{maka diperoleh persamaan (18a)}:
\]

\[
\frac{dP}{dt} = [r_i (1-i) + r_i iP(1-P) - \frac{aS}{1+bP} Z - m_i P
\]

Untuk persamaan (18b)

Persamaan (18b) diperoleh dengan menurunkan persamaan \( i \) terhadap \( t \) sebagai berikut:

\[
i = \frac{1}{P}
\]

\[
\frac{di}{dt} = \frac{di}{dt} \frac{P - i}{di} \frac{dP}{dt} \frac{1}{P^2}
\]
\[ 
\begin{align*}
\frac{dr_i}{dt} &= \left[ (r_i - r_j)(1 - P) + (\gamma - m_{ij}) \right](1 - i) \\
\text{Untuk persamaan (18c)}
\end{align*}
\]

Persamaan 18c diperoleh dengan menubhitisuskan persamaan \( P = S + I \) ke persamaan (18c) sehingga didapat:

\[ 
\frac{dZ}{dt} = \frac{a(S + I)}{1 + b(S + I)} Z - m_{ij}Z \quad \Rightarrow \quad \frac{dZ}{dt} = \frac{aP}{1 + bP} Z - m_{ij}Z
\]
LAMPIRAN 2

Pembuktian teorema 2

Teorema 2. Misalkan \( A, B, C \) bilangan-bilangan real. Bagian real dari setiap nilai eigen persamaan karakteristik

\[
p(\lambda) = \lambda^3 + A\lambda^2 + B\lambda + C = 0
\]

adalah negatif jika dan hanya jika \( A, C \) positif dan \( AB > C \).

Bukti:

Dari persamaan \( p(\lambda) = \lambda^3 + A\lambda^2 + B\lambda + C \), maka

\[a_0 = 1, a_1 = A, a_2 = B, a_3 = C \quad \text{dan} \quad a_4 = 0 \quad \text{jika} \quad i \quad \text{selainnya. Berdasarkan kriteria Routh-Hurwitz, maka}
\]

bagian real dari setiap akar polinomial \( p(\lambda) = \lambda^3 + A\lambda^2 + B\lambda + C \) adalah negatif jika dan hanya jika \( |M_1|, |M_2|, |M_3| \) positif, dimana :

\[
|M_1| = |a_1| = |A| = A > 0 \quad (1)
\]

\[
|M_2| = \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix} = \begin{vmatrix} A & C \\ 1 & B \end{vmatrix} = AB - C > 0 \quad (2)
\]

\[
|M_3| = \begin{vmatrix} a_1 & a_3 & 0 \\ 1 & a_2 & 0 \\ 0 & a_1 & a_3 \end{vmatrix} = \begin{vmatrix} A & C & 0 \\ 1 & B & 0 \\ 0 & A & C \end{vmatrix} = ABC - C^2 > 0 \quad (3)
\]

Dari (1) maka diperoleh \( A > 0 \)
Dari (2) maka diperoleh \( AB - C > 0 \)
Dari (3) maka diperoleh \( ABC - C^2 > 0 \) yang dapat diubah dalam bentuk \( C(AB - C) > 0 \), sehingga dari (2) diperoleh nilai \( C > 0 \).

Dengan demikian diperoleh bahwa bagian real dari setiap akar polinomial \( p(\lambda) = \lambda^3 + A\lambda^2 + B\lambda + C \) adalah negatif jika dan hanya jika \( A > 0, C > 0 \) serta \( AB > C \).

Terbukti
LAMPIRAN 3

Program 1. Menentukan titik tetap beserta analisis kestabilan pada titik tetap

```mathematica
intreset;
plotreset;
setstate[{P, i, Z}];
setparm[{r1, r2, \(\gamma\), m2, m3, a, b}];
slopevec = {(r1 (1 - i)) (1 - P) - \(\frac{a P}{1 + b P}\) * Z - m2 * i * P,
            ((-r1) (1 - P) + (\(\gamma\) - m2)) (1 - i) i, \(\frac{a P}{1 + b P}\) * Z - m3 * Z};
equil = findpolyeq;
{Titik tetap 1} eqfrel = equil[[1]]
{0, 0, 0}
{Titik tetap 2} eqfree2 = equil[[2]]
{0, 1, 0}
{Titik tetap 3} eqfree3 = equil[[3]]
{1, 0, 0}
{Titik tetap 4} eqfree4 = equil[[4]]
{-m3 + r1 - \(\gamma\), \(-m2 + \gamma\), r1}
{Titik tetap 5} eqfree5 = equil[[5]]
{-m3, -m2, \(\frac{a}{a - b m3}\)}
{Titik tetap 6} eqend = equil[[6]]
{-m3, \(\frac{a - m3 - b m3}{a - b m3}\), r1}
{Pemasukan Nilai Parameter}
parmval = {{1, 0, 0.8, 0.14, 0.625, 1, 1}};
{Analisis titik tetap 1}
{Analisis tetap 1 titik}
eqfreeval = eqstateval[eqfrel]
{0, 0, 0}
eigsys[eqfree1]
{{1, -0.625, -0.34}, {{1., 0., 0.}, {0., 0.1., 0.}, {0., 1., 0.}}} classif[y][eqfree1]
unstable
classify2D[eqfree1]
Abbreviations used in classify2D.
L = linear, NL = nonlinear, R2 = repeated root.
Z1 = one zero root, Z2 = two zero roots.
This message printed once.
```
unstable - spiral
\begin{align*}
\text{Analisis titik tetap2} \\
\text{Analisis tetap2 titik} \\
eq \text{freeval}=\text{eqstateval}[\text{eqfree2}] \\
\{0,1,0\} \\
eq \text{sys}[\text{eqfree2}] \\
\begin{pmatrix}
-0.625, 0.34, -0.14 \\
0, 1, 0 \\
1, 0, 0
\end{pmatrix} \\
\text{classify}[\text{eqfree2}] \\
\text{unstable} \\
\text{classify2D}[\text{eqfree2}] \\
\text{unstable - spiral}
\end{align*}
\begin{align*}
\text{Analisis titik tetap3} \\
\text{Analisis tetap3 titik} \\
eq \text{freeval}=\text{eqstateval}[\text{eqfree3}] \\
\{1,0,0\} \\
eq \text{sys}[\text{eqfree3}] \\
\begin{pmatrix}
-1, 0.66, -0.125 \\
1, 0, 0 \\
0.808243 +0. i, -0.0884016+0.582175 i, 0. +0. i
\end{pmatrix} \\
\text{classify}[\text{eqfree3}] \\
\text{unstable} \\
\text{classify2D}[\text{eqfree3}] \\
\text{strictly stable - spiral}
\end{align*}
\begin{align*}
\text{Analisis titik tetap4} \\
\text{Analisis tetap4 titik} \\
eq \text{freeval}=\text{eqstateval}[\text{eqfree4}] \\
\{0.34, 0.825, 0\} \\
eq \text{sys}[\text{eqfree4}] \\
\begin{pmatrix}
-0.371269, -0.02975+0.195921 \iota, -0.02975-0.195921 \iota \\
0.509037, -0.197949, 0.837674 \\
0.808243 +0. i, -0.0884016-0.582175 i, 0. +0. i
\end{pmatrix} \\
\text{classify}[\text{eqfree4}] \\
\text{strictly stable} \\
\text{classify2D}[\text{eqfree4}] \\
\text{strictly stable - saddle}
\end{align*}
\begin{align*}
\text{Analisis titik tetap5} \\
\text{Analisis tetap5 titik} \\
eq \text{freeval}=\text{eqstateval}[\text{eqfree5}] \\
\{1.66667, 1, -0.37333\} \\
eq \text{sys}[\text{eqfree5}] \\
\begin{pmatrix}
-1.32667, -0.230101, 0.146201 \\
-0.585634, 0.810244, -0.0231752 \\
0.938422, 0., -0.345491
\end{pmatrix} \\
\text{classify}[\text{eqfree5}] \\
\text{unstable} \\
\text{classify2D}[\text{eqfree5}] \\
\text{strictly stable - node}
\end{align*}
\begin{align*}
\text{Analisis titik tetap6} \\
\text{Analisis tetap6 titik} \\
eq \text{freeval}=\text{eqstateval}[\text{eqgend}] \\
\{1.66667, 0, -1.7777\} \\
eq \text{sys}[\text{eqgend}] \\
\begin{pmatrix}
-2.15581, 1.32667, 0.0724785 \\
-0.993343, 0., -0.115194 \\
0.257319, 0.965109, -0.0484897 \\
0.278448, 0., -0.960451
\end{pmatrix} \\
\text{classify}[\text{eqgend}] \\
\text{unstable} \\
\text{classify2D}[\text{eqgend}] \\
\text{unstable - saddle}
\end{align*}
Program 2. Analisis kestabilan dengan metode Routh-Hurwitz Criterion

Clear[r1,i,P,a,b,Z,γ,m2,m3]

eqonerhs = (r1 (1-i)) (1-P) P - \frac{a*P-Z}{1+b*P} * Z - m2 * i * P

-eqtworhs = ((-r1) (1-P) + (γ-m2)) (1-i) i

-eqtreerhs = \frac{a*P}{1+b*P} * Z - m3 * Z

-cps=Solve[{eqonerhs==0,eqtworhs==0,eqtreerhs==0},{P,i,Z}]

linmatrix={{D[eqonerhs,P],D[eqonerhs,i],D[eqonerhs,Z]},{D[eqtworhs,P],D[eqtworhs,i],D[eqtworhs,Z]},{D[eqtreerhs,P],D[eqtreerhs,i],D[eqtreerhs,Z]}};

Matrix Jacobi

\{\frac{m2+r1-γ}{r1}, \frac{r1(γ-m2)}{r1γ}, 0\}\
\[ \text{empat} = \text{linmatrix} / \{ P \rightarrow \frac{m2 + r1 - \gamma}{r1}, i \rightarrow \frac{r1 (\gamma - m2)}{(r1) \gamma}, Z \rightarrow 0 \} \]

\[ \{ - \frac{m2 (-m2 + \gamma)}{\gamma} + r1 \left( 1 - \frac{m2 + r1 - \gamma}{r1} \right) \left( 1 - \frac{-m2 + \gamma}{\gamma} \right) \]

\[ - (m2 + r1 - \gamma) \left( 1 - \frac{-m2 + \gamma}{\gamma} \right), - \frac{m2 (m2 + r1 - \gamma)}{r1} \]

\[ - \left( 1 - \frac{m2 + r1 - \gamma}{r1} \right) \frac{(m2 + r1 - \gamma)}{1 - \frac{m2 + r1 - \gamma}{r1}}, - \frac{a (m2 + r1 - \gamma)}{r1 \left( 1 + \frac{b (m2 + r1 - \gamma)}{r1} \right)} \} , \]

\[ \frac{r1 (-m2 + \gamma) \left( 1 - \frac{-m2 + \gamma}{r1} \right)}{\gamma} \}

\[ + \left( -m2 - r1 \left( 1 - \frac{m2 + r1 - \gamma}{r1} \right) + \gamma \right) \left( 1 - \frac{-m2 + \gamma}{\gamma} \right), 0 \}, \{ 0, 0, -m3 + \frac{a (m2 + r1 - \gamma)}{r1 \left( 1 + \frac{b (m2 + r1 - \gamma)}{r1} \right)} \} \}

\{ \text{Matriks} \}

\text{empat} // \text{MatrixForm}

\{ \text{Matriks} \}

\[ \left( \begin{array}{cccc}
- \frac{m2 (-m2 + \gamma)}{\gamma} + r1 \left( 1 - \frac{m2 + r1 - \gamma}{r1} \right) \left( 1 - \frac{-m2 + \gamma}{\gamma} \right) & - \frac{m2 (m2 + r1 - \gamma)}{r1} & - (1 - \frac{m2 + r1 - \gamma}{r1}) & - \frac{a (m2 + r1 - \gamma)}{r1 \left( 1 + \frac{b (m2 + r1 - \gamma)}{r1} \right)} \\
- (m2 + r1 - \gamma) \left( 1 - \frac{-m2 + \gamma}{\gamma} \right) & \frac{r1 (-m2 + \gamma) \left( 1 - \frac{-m2 + \gamma}{r1} \right)}{\gamma} & \frac{(-m2 + \gamma) \left( 1 - \frac{-m2 + \gamma}{r1} \right) \gamma + \left(-m2 - r1 \left( 1 - \frac{m2 + r1 - \gamma}{r1} \right) + \gamma \right) \left( 1 - \frac{-m2 + \gamma}{\gamma} \right)}{\gamma} & 0 \\
0 & 0 & \frac{-m3 + a (m2 + r1 - \gamma)}{r1 \left( 1 + \frac{b (m2 + r1 - \gamma)}{r1} \right)} & 0 \\
\end{array} \right) \]

\text{all} = \text{empat}[[1, 1]]; \text{a12} = \text{empat}[[1, 2]]; \text{a13} = \text{empat}[[1, 3]]; \text{a21} = \text{empat}[[2, 1]]; \text{a22} = \text{empat}[[2, 2]]; \text{a23} = \text{empat}[[2, 3]]; \text{a31} = \text{empat}[[3, 1]]; \text{a32} = \text{empat}[[3, 2]]; \text{a33} = \text{empat}[[3, 3]]; \text{Clear[A2, B2, C2]}
\[
\{\text{Jika nilai-nilai eigen dari persamaan karakteristik dapat dibentuk : } \lambda^3 + A_2 \lambda^2 + B_2 \lambda + C_2 = 0 \text{ maka :}
\}
\]

\[A_2 = -(a_{11} + a_{22} + a_{33})\]

\[m_3 - \frac{a \left(m_2 + r_1 - \gamma\right)}{r_1 \left(1 + b \frac{(m_2 + r_1 - \gamma)}{r_1}\right)} + \frac{m_2 \left(-m_2 + \gamma\right)}{\gamma} - \frac{(m_2 + \gamma) \left(-m_2 - r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right) + \gamma\right)}{\gamma} - \]

\[r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right) \left(1 - \frac{-m_2 + \gamma}{\gamma}\right) + (m_2 + r_1 - \gamma) \left(1 - \frac{-m_2 + \gamma}{\gamma}\right) - \frac{(m_2 - r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right) + \gamma) \left(1 - \frac{-m_2 + \gamma}{\gamma}\right)}{\gamma}\]

\[B_2 = -(a_{13} a_{31} + a_{32} a_{23} + a_{12} a_{21} - a_{11} a_{33} - a_{11} a_{22} - a_{22} a_{33})\]

\[r_1 \left(-\frac{m_2 (m_2 + r_1 - \gamma)}{r_1} - \left(1 - \frac{m_2 (m_2 + r_1 - \gamma)}{r_1}\right) (m_2 + r_1 - \gamma) \right) - \frac{(m_2 + \gamma) (1 - \frac{-m_2 + \gamma}{\gamma})}{\gamma} + \]

\[
\left(-m_3 + \frac{a \left(m_2 + r_1 - \gamma\right)}{r_1 \left(1 + b \frac{(m_2 + r_1 - \gamma)}{r_1}\right)}\right) \left(-\frac{m_2 \left(-m_2 + \gamma\right)}{\gamma} + \frac{1 - \frac{m_2 + r_1 - \gamma}{r_1}}{r_1}\right) \left(1 - \frac{-m_2 + \gamma}{\gamma}\right) + \frac{(m_2 + \gamma) \left(-m_2 - r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right) + \gamma\right)}{\gamma} + \]

\[
\left(-m_3 + \frac{a \left(m_2 + r_1 - \gamma\right)}{r_1 \left(1 + b \frac{(m_2 + r_1 - \gamma)}{r_1}\right)}\right) \left(-\frac{(m_2 + \gamma) \left(-m_2 - r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right) + \gamma\right)}{\gamma} + \frac{1 - \frac{m_2 + r_1 - \gamma}{r_1}}{r_1} \left(1 - \frac{-m_2 + \gamma}{\gamma}\right)\right) + \]

\[
- \frac{m_2 \left(-m_2 + \gamma\right)}{r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right)} + \frac{1 - \frac{m_2 + r_1 - \gamma}{r_1}}{r_1} \left(1 - \frac{-m_2 + \gamma}{\gamma}\right) + (m_2 + r_1 - \gamma) \left(1 - \frac{-m_2 + \gamma}{\gamma}\right)\]

\[
\left(-\frac{m_2 + \gamma) \left(-m_2 - r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right) + \gamma\right)}{\gamma} + \frac{1 - \frac{m_2 + r_1 - \gamma}{r_1}}{r_1} \left(1 - \frac{-m_2 + \gamma}{\gamma}\right)\right) + \]

\[
\left(-\frac{m_2 + \gamma) \left(-m_2 - r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right) + \gamma\right)}{\gamma} + \frac{1 - \frac{m_2 + r_1 - \gamma}{r_1}}{r_1} \left(1 - \frac{-m_2 + \gamma}{\gamma}\right)\right) + \]

\[
\left(-\frac{m_2 + \gamma) \left(-m_2 - r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right) + \gamma\right)}{\gamma} + \frac{1 - \frac{m_2 + r_1 - \gamma}{r_1}}{r_1} \left(1 - \frac{-m_2 + \gamma}{\gamma}\right)\right) + \]

\[
\left(-\frac{m_2 + \gamma) \left(-m_2 - r_1 \left(1 - \frac{m_2 + r_1 - \gamma}{r_1}\right) + \gamma\right)}{\gamma} + \frac{1 - \frac{m_2 + r_1 - \gamma}{r_1}}{r_1} \left(1 - \frac{-m_2 + \gamma}{\gamma}\right)\right)\]

\[C_2 = -(a_{11} a_{22} a_{33} + a_{13} a_{21} a_{32} + a_{12} a_{23} a_{31} - a_{32} a_{23} a_{11} + a_{13} a_{22} a_{31} - a_{12} a_{21} a_{33})\]
\[ r_1 \left( \frac{m_2 r_1 - \gamma}{r_1} - \left(1 - \frac{m_2 r_1 - \gamma}{r_1} \right) (m_2 + r_1 - \gamma) \right) \left( \frac{-m_3 + \frac{a (m_2 r_1 - \gamma)}{r_1 (1 + \frac{b (m_2 r_1 - \gamma)}{r_1})}}{\gamma} \right) \left( \frac{-m_2 r_1 - \gamma}{{r_1}} \right) \left( 1 - \frac{m_2 + \gamma}{\gamma} \right) \]
\[
\begin{align*}
\left\{ m_3 - \frac{a (m_2 + r_1 - \gamma)}{r_1 (1 + \frac{b (m_2 + r_1 - \gamma)}{r_1})} \right. & + \frac{m_2 (-m_2 + \gamma)}{\gamma} + \\
& \left. \frac{(-m_2 + \gamma) (-m_2 - r_1 (1 - \frac{m_2 + r_1 - \gamma}{r_1}) + \gamma)}{\gamma} \right.
\end{align*}
\]

\[
\begin{align*}
&-r_1 \left( 1 - \frac{m_2 + r_1 - \gamma}{r_1} \right) \left( 1 - \frac{-m_2 + \gamma}{\gamma} \right) + (m_2 + r_1 - \gamma) \left( 1 - \frac{-m_2 + \gamma}{\gamma} \right) \\
&+ \left( -m_2 - r_1 \left( 1 - \frac{m_2 + r_1 - \gamma}{r_1} \right) + \gamma \right) \left( 1 - \frac{-m_2 + \gamma}{\gamma} \right) \\
&+ \left( \frac{-m_2 + \gamma}{\gamma} + r_1 \left( 1 - \frac{m_2 + r_1 - \gamma}{r_1} \right) \left( 1 - \frac{-m_2 + \gamma}{\gamma} \right) - (m_2 + r_1 - \gamma) \left( 1 - \frac{-m_2 + \gamma}{\gamma} \right) \right)
\end{align*}
\]

\[
\begin{align*}
&-\frac{r_1 (\frac{m_2 (m_2 + r_1 - \gamma)}{r_1} - (1 - \frac{m_2 + r_1 - \gamma}{r_1}) (m_2 + r_1 - \gamma) (-m_2 + \gamma) (1 - \frac{-m_2 + \gamma}{\gamma})}{\gamma} \\
&+ \frac{(-m_2 + \gamma) (-m_2 - r_1 (1 - \frac{m_2 + r_1 - \gamma}{r_1}) + \gamma)}{\gamma}
\end{align*}
\]

\[
\left\{ \text{Masukkan titik tetap E5} \left( \frac{m_3}{a - b m_3}, 1, \frac{a (-m_2)}{(a - b m_3)^2} \right) \right\}
\]
\[
\begin{align*}
\text{lima} &= \text{limatrix/. \{P} \rightarrow \frac{m_3}{a - b m_3}^{2}, \ i \rightarrow 1, \ Z \rightarrow \frac{a (-m_2)}{(a - b m_3)^2} \}
\end{align*}
\]

\[
\begin{align*}
\left\{ -m_2 - \frac{a^2 b m_2 m_3}{(a - b m_3)^3 \left(1 + \frac{b m_3}{a - b m_3}\right)^2} + \frac{a^2 m_2}{(a - b m_3)^2 \left(1 + \frac{b m_3}{a - b m_3}\right)^2}, -m_2 - \frac{m_3 \left(1 - \frac{m_3}{a - b m_3}\right)}{a - b m_3} \right\}, \begin{array}{c} m_2 - \frac{m_2 m_3}{a - b m_3} - \frac{m_3 \left(1 - \frac{m_3}{a - b m_3}\right) r_1}{a - b m_3}, - \frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)} \end{array}
\end{align*}
\]

\[
\begin{align*}
\{0, m_2 + (1 - \frac{m_3}{a - b m_3}) r_1 - \gamma, 0\}, \begin{array}{c}
\frac{a^2 b m_2 m_3}{(a - b m_3)^3 \left(1 + \frac{b m_3}{a - b m_3}\right)^2} - \frac{a^2 m_2}{(a - b m_3)^2 \left(1 + \frac{b m_3}{a - b m_3}\right)^2}, 0, -m_3 + \frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)} \end{array}
\end{align*}
\]

\{Matriks\}
\text{lima/.MatrixForm}
\{Matriks\}

\[
\begin{align*}
\left\{ -m_2 - \frac{a^2 b m_2 m_3}{(a - b m_3)^3 \left(1 + \frac{b m_3}{a - b m_3}\right)^2} + \frac{a^2 m_2}{(a - b m_3)^2 \left(1 + \frac{b m_3}{a - b m_3}\right)^2}, -m_2 - \frac{m_3 \left(1 - \frac{m_3}{a - b m_3}\right) r_1}{a - b m_3}, - \frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)} \right\}
\end{align*}
\]

\[
\begin{align*}
0, m_2 + (1 - \frac{m_3}{a - b m_3}) r_1 - \gamma, 0, -m_3 + \frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)} \end{align*}
\]

\[
{\text{all}=\text{lima}[[1,1]]; a_{12}=\text{lima}[[1,2]]; a_{13}=\text{lima}[[1,3]]; a_{21}=\text{lima}[[2,1]]; a_{22}=\text{lima}[[2,2]]; a_{23}=\text{lima}[[2,3]]; a_{31}=\text{lima}[[3,1]]; a_{32}=\text{lima}[[3,2]]; a_{33}=\text{lima}[[3,3]]; \text{Clear}[A3, B3, C3]}
\]

\{Jika nilai - nilai eigen dari persamaan karakteristik dapat dibentuk : \( \lambda^3 + A_3 \lambda^2 + B_3 \lambda + C_3 = 0 \) maka : \}
\]

\[
\begin{align*}
A_3 &= -(a_{11}+a_{22}+a_{33})
\end{align*}
\]

\[
\begin{align*}
m_3 + \frac{a^2 b m_2 m_3}{(a - b m_3)^3 \left(1 + \frac{b m_3}{a - b m_3}\right)^2} - \frac{a^2 m_2}{(a - b m_3)^2 \left(1 + \frac{b m_3}{a - b m_3}\right)^2} - \frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)} - \left(1 - \frac{m_3}{a - b m_3}\right) r_1 + \gamma
\end{align*}
\]

\[
B_3 = -(a_{13}+a_{32}+a_{23}+a_{12}-a_{11}+a_{33}-a_{11}+a_{22}-a_{22}+a_{33})
\]
\[
\frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)} \left(\frac{a^2 b m_2 m_3}{(a - b m_3)^3 \left(1 + \frac{b m_3}{a - b m_3}\right)^2} - \frac{a^2 m_2}{(a - b m_3)^2 \left(1 + \frac{b m_3}{a - b m_3}\right)}\right)
\]

\[
+ \left(-2m_2 - \frac{a^2 b m_2 m_3}{(a - b m_3)^3 \left(1 + \frac{b m_3}{a - b m_3}\right)^2} + \frac{a^2 m_2}{(a - b m_3)^2 \left(1 + \frac{b m_3}{a - b m_3}\right)}\right)
\]

\[
\left(-m_3 + \frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)}\right) \left(-m_3 + \frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)}\right) \left(m_2 + \left(1 - \frac{m_3}{a - b m_3}\right) r_{1-\gamma}\right)
\]

\[
C_3 = -(a_{11} a_{22} a_{33} + a_{13} a_{21} a_{32} + a_{12} a_{23} a_{31} - a_{32} a_{23} a_{12} - a_{13} a_{21} a_{32} - a_{12} a_{21} a_{33})
\]

\[
a m_3 \left(\frac{a^2 b m_2 m_3}{(a - b m_3)^3 \left(1 + \frac{b m_3}{a - b m_3}\right)^2} - \frac{a^2 m_2}{(a - b m_3)^2 \left(1 + \frac{b m_3}{a - b m_3}\right)}\right) \left(m_2 + \left(1 - \frac{m_3}{a - b m_3}\right) r_{1-\gamma}\right)
\]

\[
- \frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)} \left(-m_3 + \frac{a m_3}{(a - b m_3) \left(1 + \frac{b m_3}{a - b m_3}\right)}\right) \left(m_2 + \left(1 - \frac{m_3}{a - b m_3}\right) r_{1-\gamma}\right)
\]

\[
A_3 B_3 - C_3 > 0
\]
\[
\frac{am^3}{(a - bm^3)(1 + \frac{bm^3}{a - bm^3})} \left( m^2 + (1 - \frac{m^3}{a - bm^3}) r1 - \gamma \right)
\]

\[
+ \left( -m^2 - \frac{a^2 b m^2}{(a - bm^3)^2 (1 + \frac{bm^3}{a - bm^3})^2} + \frac{a^2 m^2}{(a - bm^3) (1 + \frac{bm^3}{a - bm^3})} \right)
\]

\[
- \frac{am^3}{(a - bm^3)(1 + \frac{bm^3}{a - bm^3})} \left( m^2 + (1 - \frac{m^3}{a - bm^3}) r1 - \gamma \right)
\]

\[
+ \left( -m^2 - \frac{a^2 b m^2}{(a - bm^3)^2 (1 + \frac{bm^3}{a - bm^3})^2} + \frac{a^2 m^2}{(a - bm^3) (1 + \frac{bm^3}{a - bm^3})} \right)
\]

\[
- \frac{am^3}{(a - bm^3)(1 + \frac{bm^3}{a - bm^3})} + \left( -m^2 - \frac{a^2 b m^2}{(a - bm^3)^2 (1 + \frac{bm^3}{a - bm^3})^2} + \frac{a^2 m^2}{(a - bm^3) (1 + \frac{bm^3}{a - bm^3})} \right)
\]
\[
\begin{align*}
(m_2 + (1 - \frac{m_3}{a - b m_3}) r_1 - \gamma) + & \left\{ -m_3 + \frac{a m_3}{(a - b m_3) (1 + \frac{b m_3}{a - b m_3})} \right\} \\
(m_2 + (1 - \frac{m_3}{a - b m_3}) r_1 - \gamma) m_3 + & \left\{ m_3 + \frac{a^2 b m_2 m_3}{(a - b m_3)^3 (1 + \frac{b m_3}{a - b m_3})^2} - \frac{a^2 m_2}{(a - b m_3)^2 (1 + \frac{b m_3}{a - b m_3})} \right\} \\
- & \frac{a m_3}{(a - b m_3) (1 + \frac{b m_3}{a - b m_3})} - \left(1 - \frac{m_3}{a - b m_3} \right) r_1 + \gamma \right\} > 0
\end{align*}
\]

{Masukkan titik tetap E6 \( \left\{ \frac{m_3}{a - b m_3}, 0, \frac{(a - (1 + b) m_3) r_1}{(a - b m_3)^2} \right\} \)}
enam = LinMatrix /.
\[
\begin{align*}
\{ & [- \frac{m_3 r_1}{a - b m_3} + (1 - \frac{m_3}{a - b m_3}) r_1 \\
& + \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 (1 + \frac{b m_3}{a - b m_3})^2} - \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 (1 + \frac{b m_3}{a - b m_3})} , \\
& - \frac{m_2 m_3}{a - b m_3} - \frac{m_3 (1 - \frac{m_3}{a - b m_3})}{a - b m_3} , - \frac{a m_3}{(a - b m_3) (1 + \frac{b m_3}{a - b m_3})} \},
\end{align*}
\]

\{0, -m_2 - (1 - \frac{m_3}{a - b m_3}) r_1 + \gamma, 0\}, \left\{ - \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 (1 + \frac{b m_3}{a - b m_3})^2} + \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 (1 + \frac{b m_3}{a - b m_3})} , 0, -m_3 + \frac{a m_3}{(a - b m_3) (1 + \frac{b m_3}{a - b m_3})} \} \}

{Matriks}
enam//MatrixForm
\[
\{ \text{Matriks} \}
\]

\[
\begin{pmatrix}
-m_3 r_1 + \frac{m_3}{a-b m_3} + (1 - \frac{m_3}{a-b m_3}) r_1 + \frac{a b m_3 (a-(1+b) m_3) r_1}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} - \frac{a (a-(1+b) m_3) r_1}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} r_1 - \frac{m_3 (1-\frac{m_3}{a-b m_3}) r_1}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} - \frac{a m_3}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} \cr 0 \cr -\frac{a b m_3 (a-(1+b) m_3) r_1}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} + \frac{a (a-(1+b) m_3) r_1}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} r_1 - \frac{m_3}{a-b m_3} (1-\frac{m_3}{a-b m_3}) r_1 + \gamma \cr 0 \cr -\frac{m_3 r_1}{a-b m_3} + \frac{m_3}{a-b m_3} + (1 - \frac{m_3}{a-b m_3}) r_1 + \frac{a b m_3 (a-(1+b) m_3) r_1}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} - \frac{a (a-(1+b) m_3) r_1}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} r_1 - \frac{m_3 (1-\frac{m_3}{a-b m_3}) r_1}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} - \frac{a m_3}{(a-b m_3)^2 (1+\frac{b m_3}{a-b m_3})^2} \cr
\end{pmatrix}
\]

\[a_{11} = \text{enam}[1,1]; a_{12} = \text{enam}[1,2]; a_{13} = \text{enam}[1,3]; a_{21} = \text{enam}[2,1]; a_{22} = \text{enam}[2,2]; a_{23} = \text{enam}[2,3]; a_{31} = \text{enam}[3,1]; a_{32} = \text{enam}[3,2]; a_{33} = \text{enam}[3,3]; \text{Clear}[A_{4}, B_{4}, C_{4}]
\]

\{Jika nilai-nilai eigen dari persamaan karakteristik dapat dibentuk : \( \lambda^3 + A_{4} \lambda^2 + B_{4} \lambda + C_{4} = 0 \) maka:

\[A_{4} = -(a_{11} + a_{22} + a_{33})
\]

\[B_{4} = -(a_{13} * a_{31} + a_{23} * a_{32} + a_{21} - a_{11} * a_{33} - a_{11} * a_{22} - a_{22} * a_{33})
\]

\[C_{4} = -(a_{11} * a_{22} + a_{13} * a_{21} + a_{12} * a_{23} - a_{13} * a_{22} + a_{21} - a_{23} * a_{33})
\]
\[
\left( -m_3 \cdot \frac{a m_3}{(a - b m_3) \left( 1 + \frac{b m_3}{a - b m_3} \right)} \right) \cdot \left( \frac{m_3 r_1}{a - b m_3} + \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 + \right.
\\
\left. \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 \left( 1 + \frac{b m_3}{a - b m_3} \right)^2} - \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 \left( 1 + \frac{b m_3}{a - b m_3} \right)} \right)
\\
(-m_2 - \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 + \gamma)
\\
\left( a m_3 \left\{ \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 \left( 1 + \frac{b m_3}{a - b m_3} \right)^2} + \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 \left( 1 + \frac{b m_3}{a - b m_3} \right)} \right\} (-m_2 - \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 + \gamma)
\\
\right)
\\
(a - b m_3) \left( 1 + \frac{b m_3}{a - b m_3} \right)
\\
A_4 \cdot B_4 - C_4 > 0
\]
\[
\begin{align*}
&\left\{ -m_3 + \frac{a m_3}{(a - b m_3) \left( 1 + \frac{b m_3}{a - b m_3} \right)} \right. \\
&\left. - \frac{m_3 r_1}{a - b m_3} + \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 \right. \\
&\phantom{\left( \right)} + \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 \left( 1 + \frac{b m_3}{a - b m_3} \right)^2} - \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 \left( 1 + \frac{b m_3}{a - b m_3} \right)} \\
&\phantom{\left( \right)} - m_2 - \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 + \gamma \\
&\phantom{\left( \right)} + a m_3 \left\{ - \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 \left( 1 + \frac{b m_3}{a - b m_3} \right)^2} + \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 \left( 1 + \frac{b m_3}{a - b m_3} \right)} \right\} (-m_2 - \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 + \gamma) \\
&\phantom{\left( \right)} \left( a - b m_3 \right) \left( 1 + \frac{b m_3}{a - b m_3} \right) \\
&\left\{ m_2 + m_3 - \frac{a m_3}{(a - b m_3) \left( 1 + \frac{b m_3}{a - b m_3} \right)} + \frac{m_3 r_1}{a - b m_3} \\
&\phantom{\left( \right)} - \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 \left( 1 + \frac{b m_3}{a - b m_3} \right)^2} + \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 \left( 1 + \frac{b m_3}{a - b m_3} \right)} - \gamma \right\}
\end{align*}
\]
\[
\begin{align*}
\left( -m_3 + \frac{a m_3}{(a - b m_3) (1 + \frac{b m_3}{a - b m_3})} \right) \left( -m_3 r_1 + \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 \right) \\
\quad + \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 \left( 1 + \frac{b m_3}{a - b m_3} \right)^2} - \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 \left( 1 + \frac{b m_3}{a - b m_3} \right)} \\
\quad + \frac{a m_3 \left( - \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 \left( 1 + \frac{b m_3}{a - b m_3} \right)^2} + \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 \left( 1 + \frac{b m_3}{a - b m_3} \right)} \right)}{(a - b m_3) \left( 1 + \frac{b m_3}{a - b m_3} \right)} \\
\quad + \left( -m_2 - \frac{a m_3}{(a - b m_3) \left( 1 + \frac{b m_3}{a - b m_3} \right)} \right) \left( -m_2 \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 + \gamma \right) + \\
\left( -m_3 r_1 + \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 + \frac{a b m_3 (a - (1 + b) m_3) r_1}{(a - b m_3)^3 \left( 1 + \frac{b m_3}{a - b m_3} \right)^2} - \frac{a (a - (1 + b) m_3) r_1}{(a - b m_3)^2 \left( 1 + \frac{b m_3}{a - b m_3} \right)} \right) \\
\quad \left( -m_2 \left( 1 - \frac{m_3}{a - b m_3} \right) r_1 + \gamma \right) > 0
\end{align*}
\]
Program 3. Menentukan hubungan populasi fitoplankton, populasi fitoplankton yang terinfeksi virus dan zooplankton terhadap $t$

$$a = 1;$$
$$b = 1;$$
$$r_1 = 1;$$
$$r_2 = 0;$$
$$\gamma = 0.8;$$
$$m_2 = 0.14;$$
$$m_3 = 0.625;$$
$$t_{\text{max}} = 10000;$$

`soll = NDSolve[
{P'[t] == (r_1 - i[t]) + r_2 * i[t]) (1 - P[t]) P[t] - \frac{(a * P[t])}{(1 + b * P[t])} * Z[t] - m_2 * i[t] * P[t],

i'[t] == ((r_2 - r_1) (1 - P[t]) + (\gamma - m_2) (1 - i[t]) i[t], Z'[t] == \frac{(a * P[t])}{(1 + b * P[t])} * Z[t] - m_3 * Z[t],

P[0] == 0.9, i[0] == 0.1, Z[0] == 0.3, {P[t], i[t], Z[t]}, (t, 0, t_{\text{max}}), MaxSteps \to \infty};

{phyto, vir, zoo} = {P[t], i[t], Z[t]}. Flatten[soll];

(* Memanggil grafik untuk data *)
data1 = Table[phyto, {t, 0, t_{\text{max}}}];
data2 = Table[vir, {t, 0, t_{\text{max}}}];
data3 = Table[zoo, {t, 0, t_{\text{max}}}];

(* Menampilkan grafik untuk data1 *)
graf1 = ListPlot[data1, PlotStyle -> {RGBColor[1, 0, 0], Thickness[0.008]},
AxesLabel -> {"t", "P"}, PlotJoined -> True, AspectRatio -> 1, PlotRange -> {{0, 2000}, {0, 1.2}},
ImageSize -> 300];

(* Menampilkan grafik untuk data2 *)
graf2 = ListPlot[data2, PlotStyle -> {RGBColor[0, 1, 0], Thickness[0.008]},
AxesLabel -> {"t", "i"}, PlotJoined -> True, AspectRatio -> 1, PlotRange -> {{0, 2000}, {0, 1}},
ImageSize -> 300];

(* Menampilkan grafik untuk data3 *)
graf3 = ListPlot[data3, PlotStyle -> {RGBColor[0, 0, 1], Thickness[0.008]},
AxesLabel -> {"t", "Z"}, PlotJoined -> True, AspectRatio -> 1, PlotRange -> {{0, 50}, {0, 1}},
ImageSize -> 300];

(* Menampilkan keseluruhan grafik yang ada *)
Show[{graf1, graf2, graf3}, AxesLabel -> {"t", "P,i,Z"}, DisplayFunction -> $DisplayFunction, PlotRange -> {{0, 800}, {0, 1}}, ImageSize -> 300]
Program 4. Menentukan Orbit

(* Memeriksa versi dari paket DynPac *)
sysid
Mathematica 5.2.0, DynPac 10.71, 12/27/2007

(* Mereset semua variabel bernilai integer yang ada *)
intreset;

(* Mereset semua gambar yang ada *)
plotreset;

(* Menyatakan state variabel *)
setstate[{P,i,Z}];

(* Menyatakan variable yang dibutuhkan *)
setparm[{r1,r2,m2,m3,γ,a,b}];

(* Set functions of vector field *)
slopevec = { (r1 (1-i) + r2 * i) (1-P) P - \frac{a * P}{1 + b * P} * Z - m2 * i * P,

((r2 - r1) (1-P) + (γ - m2)) (1-i) i, \frac{a * P}{1 + b * P} * Z - m3 * Z};

(* Konstanta dari parameter *)
parmval={1,0,0.14,0.625,0.8,1,1};

(* Pemberian nama sistem *)
sysname="phytozoo";

(* kondisi awal dari state variabel *)
initvec={1.01,0.1,0.001};

(* Waktu kondisi awal, biasanya nol *)
t0=0.0;

(* Selang Integral *)
window={{0,0.5},{0,0.5},{0,0.5}};

(* Kenaikan waktu berikutnya *)
h=sugtimestep[window]
0.0311318

(* Banyak langkah integral *)
nsteps=2000;

(* Perhitungan Integral *)
sol=integrate[initvec,t0,h,nsteps];

Gambar hasil simulasi 3D
boxrat={1,1,1};
graph2=phaseplot3D[sol,1,2,3];
Mentransformasikan gambar hasil simulasi 3D ke dalam 2D

(* Memastikan semua ratio perbandingan antara bidang 2D punya ukuran yang sama *)
asprat=1.0;

(* Membuat tanda panah dalam gambar *)
arrowflag=True;
(* Set arrow-vector *)
arrowvec=\left\{ \frac{1}{3} \right\};

(* Membuat rentang nilai pada gambar *)
plrange={0,1},{0,0.004};

(*Menggambar bidang 2D pada sumbu P,i *)
graphxyproj1=phaseplot[sol,1,2];

(* Membuat rentang nilai pada gambar *)
plrange={0,1},{0,0.004};

(*Menggambar bidang 2D pada sumbu P,Z *)
graphxzproj2=phaseplot[sol,1,3];

(*Membuat rentang nilai pada gambar *)
plrange={0,1},{0,0.004};

(* Menggambar bidang 2D pada sumbu i,Z *)
graphyzproj3=phaseplot[sol,2,3];