When Faustmann Forest Stand Is Facing A Convex Regeneration Cost

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Abstract

If the cost of forest regeneration is convex or contains convex segments with increasing land area that must be forested, then the Faustmann approach has the potential to not provide an optimal cycle. This happens when the available land to be planted is too small or too large. With the regeneration cost that contains the convex segment, the optimization problem is not merely looking for the optimal cycle but also the optimal stand area. If the available land is too small so that the development of plantation forests is not feasible, then allowing the land in question not to be used to plant forests is reasonable. However, if the non-feasibility is caused by the size of the land which is too large, then it is very clear that there is something that needs to be fixed. The paper shows how optimal rotation and optimal area of the stand are solved simultaneously, especially when the available land is very large so that the land expectation value becomes negative. There is a condition that makes not all available land needs to be forested if the goal to be achieved is the maximum land expectation value.

Keywords: Faustmann forest, convex regeneration costs, optimal rotation, optimal forest area, multiple Faustmann forests
JEL classification: Q28

1. Introduction

An optimal age of forest harvests is the central issue in the forest management, which was pioneered by Faustmann. A Faustmann forest is a forest that is developed by indefinitely repeated process in which the whole land is planted with seedlings initially, and all seedlings are allowed to grow to an age T (where T is a solution to the Faustmann problem), at which time the whole forest is cut down and replanted with seedlings [11].

Since it was published in 1849, the Faustmann formula has attracted so many scholars until today. The correctness of the formula was recognized and highly respected by Samuelson [12]. Implicitly, the Faustmann formula assumes that unit costs of harvesting and regeneration are independent of the cut. In other words, the unit costs are constant in the cut. Moreover, this assumption becomes common practices in forest economic studies [see 2, 7, 11, 15]. With constant unit costs per ha, then the total regeneration cost will be linear in the cut. Even though Chang [5]

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tried to generalize the Faustmann's formula by letting the stumpage price, timber yield, regeneration cost, and interest rate vary from timber crop to timber crop, still the unit costs per ha are independent of the cut.

The link between economies of scale and rotational dynamic behavior is found in [14], where the economies of scale addressed are the harvesting costs that are dependent on biomass and the ones that are dependent on harvest intensity. However, the technology involved is increasing to scale, so that regardless of how large the size of land to be planted it will not be a problem at all. As in the regular Faustmann forest case, the whole land should be planted if profitable or left idle otherwise. The situation will change dramatically if the cost function contains a segment that is convex in the size of land planted with forest. From now on, this type of cost function will be called, in short, a convex cost to emphasize the significant impact of the convexity on the optimization decision.

As suggested by [8]: "The Faustmann model provides no insight for marginal land use shifts as it holds constant the land area under management." Also, the existence of a convex cost in Forestry was stated explicitly by [3] by saying "the technology of shortwood systems showed evidence of decreasing return to scale." In addition, if transport costs outweigh scale economies, then decentralization of production to many locations is warranted [1].

A convex cost in forestry may exist in real world as [6] suggests that there is a maximum distance for trucking of woody bio-fuels. Furthermore, if we treat the maximum distance as the radius of the land managed just for simplicity of imagination, then there exist a maximum limit of forest area that can be managed economically. Between zero and the maximum acreage, it is possible that there exist an optimal area of forest that delivers the highest profit. Additionally, a concern of making transportation costs in forestry most efficient is also raised by [13], which is certainly associated with the forest area covered.

This very short article shows the case where the Faustmann solution, the maximization of the Land Expectation Value (LEV), involving the whole available land as a single unit of Faustmann forest might not be obtained due to convex regeneration costs. With a convex regeneration costs, planting the whole land available might not be profitable or optimal, but it is profitable when the planting is done partly. In this situation, what should the forest manager decide? Should the manager leave the whole land idle or should the manager plant the land part by part. The latter suggests that the problem of Faustmann forest is not merely the optimization of rotation but also the optimization of the forest size.

2. Possible Problems and Solutions

2.1. Possible Problems

On the basis of the cost curves that may be encountered, there are three general possible situations: a) Constant average cost, b) Strictly decreasing average cost, c) Strictly increasing average cost, d) Combination of those average costs. The Faustmann approach has no problem with the first two situations, but it does with the second two situations. This paper mainly deals with the second ones. Let x denote the size of land available to be planted with forest, p a constant stumpage price per unit of timber volume, t age of the forest stand, $V(t)$ marketable timber volume of the forest stand per ha as a function of the age, c a constant regeneration cost per ha of plantation, and r discount rate.

Let us start with the most common problem that is widely used in the literatures, namely constant average cost, denoted by c. The total cost is linear. There is no maximum nor minimum size of land that may be planted with forest. The available land is either planted or not planted entirely with forest. If the c is greater than the present value of one ha revenue of the mature forest ($pV(t)e^{-rt}$), then the entire available land should be planted with forest, otherwise the entire available land should not be planted with forest. The illustration of this situation is presented in Figure 1.

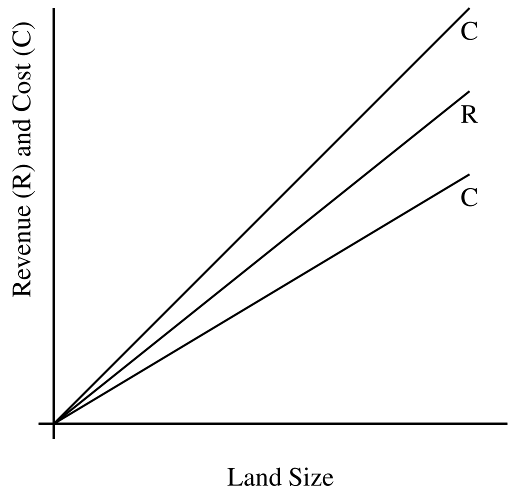

Figure 1: Constant Average Cost in Land Size

The second situation is when the average cost is strictly decreasing in the land size planted with forest. It implies that the total cost is strictly concave in the size of land planted (Figure 2). This situation has no a maximum size of the land that can be planted with forest, but may have

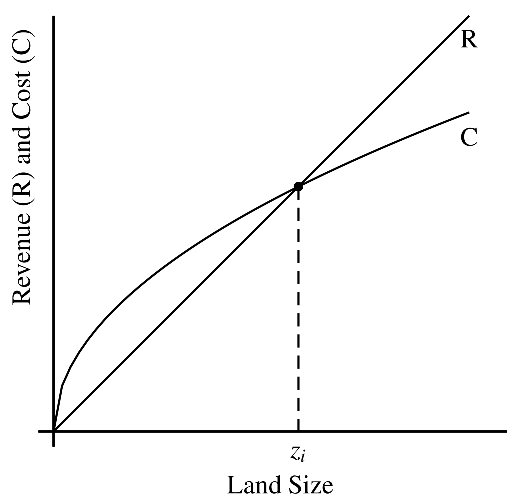

Figure 2: Decreasing Average Cost in Land Size

a minimum size of the land that forest plantation is profitable (z_i). If the land available for forest plantation is greater than z_i, then planting forest is profitable, otherwise it is not profitable.

The third situation has a maximum size of the land that can be planted with forest profitably, but has no a minimum size. If the area of the land planted with forest is larger than this maximum size (z_a), then the total revenue will be less than the total cost resulting in a loss (Figure 3). In

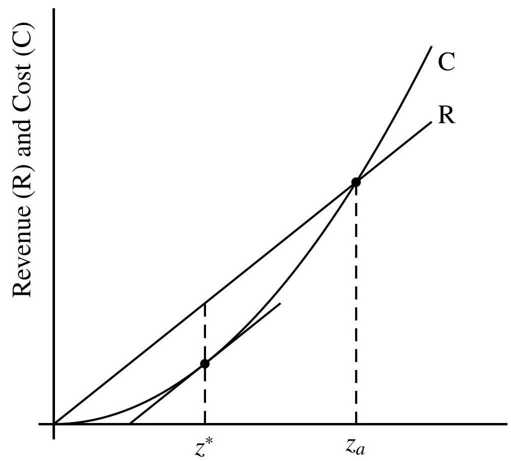

Figure 3: Increasing Average Cost in Land Size

addition, there is more important issue in this situation, namely the optimum size of land planted with forest that provides the maximum profit (z^*). The question is what should we do when the land available for forest plantation is larger than z^*; should the entire land be allocated for forest, or only z^* be allocated for forest and the rest for other profitable uses.

The fourth situation is the combination of all those previous three situations, where the cost curve has concave and convex parts in the area of land planted with forest (Figure 4). Since

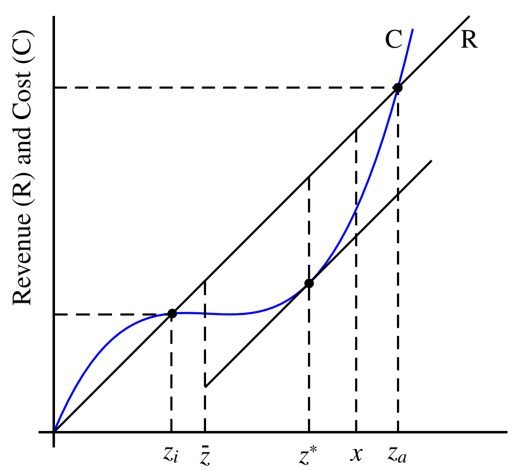

Figure 4: Revenue and Cost

between the land area planted with forest and the timber produced is positively linear, we can replace the land size with the timber quantity. In other words, the Figure 4 is standard microeconomics in the profit maximization by choosing the optimal output. However, this is a new element in the optimal rotation problem of the Faustmann tradition that usually does not take the land size into consideration. Under the fourth situation, we have problems of the minimum and maximum plantation areas beyond which planting forest is not profitable. It implies that between the minimum and maximum plantation areas there is the size of plantation forest that yields the highest profit. The crucial question is what should be done when the land available for forest plantation has the size that much larger than the maximum forest area.

2.2. Analytical Solutions

2.2.1. The Traditional Faustmann Approach

The land expectation value (LEV) of the Faustmann forest is formulated as follows

$$\max_t \left\{ \left[xpV(t)e^{-rt} - cx \right] R \right\} \tag{1}$$

where $R = \sum_{i=1}^{\infty} e^{-rt(i-1)}$, $i = 1, \ldots, \infty$ and i is the plantation cycle. Since R is simply a geometric series, then it may be simplified into $R = \frac{1}{1 - e^{-rt}}$, so that the Equation (1) may be rewritten as

$$\max_t \left\{ \left[xpV(t)e^{-rt} - cx \right] \frac{1}{1 - e^{-rt}} \right\} \tag{2}$$

So long the quantity of $pV(t)e^{-rt} - c$ is greater than zero, then the optimal t can always be obtained. It is well known as an optimal rotation problem, where the forest manager chooses merely the optimal age to sell the forest stand. The solution of this problem is t that satisfies the following equation:

$$V'(t) = \frac{re^{rt}(pV(t) - c)}{p\left(e^{rt} - 1\right)} \tag{3}$$

2.2.2. A Modified Faustmann Approach

Basically, the decision of the Faustmann approach is take it or leave it; all available land must be planted or left unplanted. What if the cost of regeneration, $C(z)$, contains a convex segment in the size of the planted land, z ($\leq x$), so that $pV(t)e^{-rt} - C(z)$ is positive to a certain extent and then turns negative after that? The Faustmann approach does not give any room to choose the area of land that needs to be planted and the rest is left unplanted. Of course, leaving entire land unplanted while planting a portion of it still provides positive profit is not a rational economic decision. Thus, in the face of convex regeneration costs, the Faustmann forest problem is no longer merely choosing optimal rotation but also optimal forest size. In this sub-section, land, in addition to time, is treated as an input in forest production process and hence it is not necessarily used entirely. As a matter of fact, the determination of optimal rotation is in essence to decide amount of time to be used in maximizing the LEV, although availability of time itself is infinite. Of course we can do the same to land, so that an optimal size of the Faustmann forest stand can be established. In microeconomic parlance, time and land have free disposal property [10, 16].

The problems faced by Faustmann forest managers are as follows:

$$\max_{z,t} \left\{ [zpV(t)e^{-rt} - C(z)] \frac{1}{1 - e^{-rt}} \right\} \quad \text{where} \quad z \leq x \tag{4}$$

The optimal rotation and optimal forest area are determined simultaneously using the following two equations:

$$V'(t) = \frac{re^{rt}\left(pV(t) - \frac{C(z)}{z}\right)}{p\left(e^{rt} - 1\right)} \tag{5}$$

and

$$pe^{-rt}V(t) = C'(z) \tag{6}$$

As can be seen, in Equation (5) the optimal rotation is influenced by the forest area, while in Equation (6) the optimal forest area is affected by the forest age. In Equation (5), the optimal rotation becomes longer when the average regeneration cost increases. While the average regeneration cost is determined by the convexity of the regeneration cost. Thus, the convexity of regeneration cost determines the optimal rotation.

Meanwhile, the effect of forest age on optimal forest area cannot be determined definitively as a result of the conflicting influences of the age in the quantity of $e^{-rt}V(t)$; a longer age will decrease the magnitude of e^{-rt}, which is also influenced by the discount rate, r, but increases the magnitude of $V(t)$. In other words, the net effect of increasing forest age on the optimal forest area cannot be determined with certainty. Hence, the optimal rotation and the optimal size of forest should be determined simultaneously.

3. Discussion

3.1. Special Case

As long as the average cost of forest regeneration is constant, the faustmann approach will provide two possibilities, namely to plant all available land or not plant the available land at all. The introduction of convex forest regeneration costs to Faustmann forests raises at least three further questions. First, if quantity $pV(t)e^{-rt} - \frac{C(z)}{z} > 0$ is positive, is planting all available land with forests definitely an optimal choice? Of course not necessarily because the optimal forest area, z^*, may be smaller than the available land. If this is the case, then the optimal rotation, t^*, obtained will be larger than it should be. This case implies that the average regeneration costs is higher than it should be. Using the Equation (3), one can show that $\frac{dt}{dc}$ is greater than zero [5, 4].

Second, if the optimal forest area is smaller than the available land area, what will the remaining land be treated for? Third, what if the remaining land is also planted with Faustmann forest with the same cost function but managed independently from the first Faustmann forest? This third question can be repeated if there is still land left after planting the second Faustmann forest and the repetition may be continued until the remaining piece of land has a size less than the optimal size. Now, there are two possibilities concerning with what to do with this remaining piece of land. The land will be planted with forest if this choice is profitable or the land will be left unplanted with forest otherwise. Perhaps this third question supports the justification for developing a normal forest.

Unfortunately, the convex costs of regeneration are less acceptable in the forestry realm, even at the theoretical level. Forestry scientists are still very comfortable with the average regeneration costs that are constant or decreasing with the increase in forest area planted. In fact, with the increase in land area to be planted, the average distance of movement of heavy equipment from the maintenance control center will increase, so building a new maintenance control center that is independent from other maintenance control centers may be able to reduce the cost of operating heavy equipment that can reduce the cost of forest regeneration. A real case is the state-owned forests of more than two million ha in Java Island of Indonesia that are managed by the state-owned company, namely Perhutani. Should the forests be managed under a single unit of the Faustmann style forest controlled from a central office? Of course, it is not sensible to treat the those forests as a single unit of the Faustmann style forest. In fact, the Perhutani employs multi units of a normal forest. A thorough description of normal forests was written by [9].

The convex regeneration costs could be just a segment of the whole cost function that reflects the production technology consisting of the segments of increasing return to scale, constant return to scale, and decreasing return to scale. Up to the constant return to scale the larger the size of the land to be forested is better in the sense that the efficiency is increasing. So nothing to worry along this segment from increasing the size of land to be forested. In other words, the size of land may be ignored in the optimization process so that the problem is merely to choose the optimal rotation. However, after the constant return to scale has been reached an increase in the size of land planted will lower the efficiency.

Actually, a timber production can be modelled as a regular production process with land, labor, silvicultural efforts, and time as inputs. To maintain simplicity and focus, let us assume that labor and silvicultural efforts do not enter the timber production function. Time works through biomass growth, denoted as $V(t)$. Certainly time meets free disposal property, otherwise the forest planted will never be harvested. Meanwhile, land plays as a multiplier role in the revenue side, so that the total salable biomass is simply formulated as $zV(t)$, and is the main component in the cost side. Is land meets free disposal property? Absolutely, it is, so we do not have to use the whole land available for plantation unless doing so delivers the maximum soil expectation value. Unfortunately, treating land as a free disposable input in forestry, the Faustmann approach tradition in particular, is very uncommon.

Let us consider Figure 4. The Faustmann model commonly perceived will work profitably if the land available for plantation is between z_i and z_a, including z_i and z_a themselves; less than z_i or larger than z_a will result in a negative land expectation value. If the land available for plantation is less than z_i, then it is plausible not to use the land for planting forest. We may say that the scale is not economic enough to grow forest. However, if the land available for plantation is larger than z_a, let us say t^*z^* where t^* is the optimal rotation for land area of z^*, then saying that the scale is not economic enough to grow forest is ridiculous. Instead of having a normal forest with t^* plots of z^* ha each, the forest is never materialized because it is not profitable. Indeed, the economic literature discusses the Faustmann model as if its application is general [8]. Hence, adding land area as a choice variable in maximizing the land expectation value is essential.

Let us consider Figure 4 once more. Suppose the land area available for plantation is x, where $x - z^* < z_i$. It is so clear that planting only z^* will provide higher LEV than planting the whole available land. What should the land owner choose? Should the land owner choose to plant only z^* and to let the remaining $x - z^*$ idle or for other uses? Or, should the land owner divide the land into two units with equal size, say $\bar{z}$, and develop two independent units of the Faustmann forest? Or, any other rule that can be employed by the land owner to maximize the LEV? Reallocation of the land should be allowed so long the remaining lands are able to yield non-negative profit:

$$\pi = z_r p V(t) e^{-rt} - C(z_r) \geq 0 \tag{7}$$

where $z_r = x - z^*$. Instead of having two units of Faustmann forest stand with different size, one with z^* ha and the other with z_r ha, we may have two units of Faustmann forest stand with the same size, namely $\frac{1}{2}(z^* + z_r)$ ha each.

3.2. General Case

Now, let us generalize the results of the special case to X ha of land, where $X >> x$. The first step is to obtain z^* by employing Equation (5) and (6). This step is similar with the determination of optimal plant size [1, 17]. The second step is to obtain number of units of the Faustmann forest stand, the quotient, by conducting a division of integer as follows

$$n = X \div z^* \tag{8}$$

The third step is to calculate the remainder of X divided by n, denoted by z_e as follows:

$$z_e = X \mod n \tag{9}$$

We use this remainder to decide what to do with it, namely

$$\text{If } z_e \begin{cases} < z_i, & \text{then for other uses or be idled} \\ \\ \geq z_i, & \text{then for forest plantation} \end{cases} \tag{10}$$

Finally, the fourth step is needed in the case of $z_e \geq z_i$. Here we have two choices: 1) to develop n units of z^* ha each and one unit of z_e ha, or 2) to develop $n + 1$ units of z^{**} ha each, where $z^{**} = X/(n + 1)$. Since the available lands are assumed to be uniform, the forest growth function employed is the same, and $\pi(z)$ is concave in z for $z \in [z_i, z_a]$, then the optimal allocation of land delivering the highest total profit must be characterized by

$$\frac{\partial \pi_1}{\partial z} = \cdots = \frac{\partial \pi_{n+1}}{\partial z} \tag{11}$$

which says that the marginal profits of land for all $n + 1$ units of the Faustmann forest stand are the same at the optimal. The choice that is consistent with this characteristic is the second one. Consistent with Equation (11) it follows that

$$n\pi(z^*) + \pi(z_r) \leq (n + 1)\pi(z^{**}) \tag{12}$$

4. Conclusions

Facing regeneration costs containing a convex segment with the cut, the traditional Faustmann model dealing merely with the optimal rotation for the whole land available might have no solution. If the solution do exist, the soil expectation value might not be truly being maximized. Hence, to find out better soil expectation value in the case of facing convex regeneration costs, the traditional rotation optimization should be complemented with the forest size optimization. Together, the optimal rotation and optimal forest area will constitute better soil expectation value.

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